Loading transformers with non sinusoidal currents

K Factor

1 Loading transformers with non sinusoidal currents .............................................................................. 2
2 Interpretation / example ............................................................................................................................. 6
1 Loading transformers with non sinusoidal currents

All ratings for transformers are based on sinusoidal rms values, rated power, rated voltage, losses, temperature rise, etc.

For a three phase transformer the main ratings, rated power; \( P_r \), rated voltage: \( U_r \); and rated current; \( I_r \), are linked through the well known equation:

\[
I_r = \frac{P_r}{U_r \sqrt{3}} \quad [1]
\]

Load losses and temperature rise are based on sinusoidal rms rated current.

Load losses can be expressed as follows:

\[
P_{LL} = P_{DC} + P_{WE^{50}} + P_{OS} \quad [2]
\]

Being:

- \( P_{DC} \): DC losses produced by a DC current of the same value that rms rated current.
- \( P_{WE^{50}} \): Winding eddy losses at the rated frequency produced by circulation current (eddy) in the windings due to leakage flux.
- \( P_{OS} \): Other stray losses, produced in bus bars and steel parts due to leakage flux, these losses have not any influence on winding temperature rise.

All losses referred to the reference temperature.

When transformer supply a non sinusoidal load current with the same rms value than rated current, DC losses remain constant but winding eddy losses increase due to the higher frequency of the harmonics, as a result, winding temperature rise increases and temperature limits can be exceed.

A transformer requested to supply non sinusoidal loads, shall be oversized in order to guarantee that windings temperature limits are not exceed in service. The value “K” of such over sizing factor – which means that temperature rise limits are guaranteed when test current is \( K \) times rated current – is calculated as follows:

\[
K = \left[ \frac{P_{DC} + P_{EW-H}}{P_{DC} + P_{EW-50}} \right]^{0.5} \quad [3]
\]

- \( P_{EW-H} \): Winding eddy losses at load current with harmonics.

Usually winding eddy losses are given in per unit of DC losses, that is:

\[
(pu)_{EW-50} = \frac{P_{EW-50}}{P_{DC}} \quad [4] \quad (pu)_{EW-H} = \frac{P_{EW-H}}{P_{DC}} \quad [5]
\]
Remark: For HD 538 Factor K calculation method, \(P_{EW-50}\) and \(P_{DC}\) are total eddy winding losses and total DC losses for the transformers because anything else is specified. Other standards, such as UL 1562 or IEEE C57.110 use individual winding values, calculated according to the method proposed in the applicable standard.

The following ratio,

\[
\frac{(pu)_{EW-H}}{(pu)_{EW-50}} = K_{EF-WE} = \sum_{h=1}^{h=H} I_{h}(pu)^2 h^q \approx \left[ 1 + \sum_{h=2}^{h=H} I_{h}(pu)^2 h^q \right] [6]
\]

Hereafter referred as \(K_{EF-WE}\), is the enhancement factor for winding eddy losses, being:

- \(I_{h}(pu)\) rms current for harmonic “\(h\)” in pu of rms rated current \(I_{rated}\)
- \(h\) is the harmonic number
- \(q\) Exponent for harmonic number, for transformer with foil windings in LV, \(q = 1.5\) as recommended in HD 538.

Eddy losses in conductors exposed to external flux, such as leakage flux in transformers, vary with the product (conductor width x frequency)\(^q\), \(q\) value varies between 2 for lower values of product and 0.5 for larger values. This is the reason why for foil LV windings, where leakage flux has a big radial component perpendicular to foil surface, a value equal to 1.5 is recommended.

Remark: according to the UL 1562, this exponent is equal to 2, however such value is very conservative leading to a transformer over size beyond the actual needs.

On this basis, eddy losses enhancement factor \(K_{EF-WE}\) shall be calculated using the equation [6] and considering \(q\) equals to 1.5, that is:

\[
K_{EF-WE} = \left[ 1 + \sum_{h=2}^{h=H} I_{h}(pu)^2 h^{1.5} \right] [6B]
\]

Taking into account that:

\[
I_{rated} \approx \left[ \sum_{h=1}^{h=H} I_{h}(pu)^2 \right]^{0.5} [7]
\]

Preferably, customer should specify the full harmonic content. In such case, the calculation of eddy losses enhancement factor and rated current can be done by using equations [6B] and [7].
Alternatively, customer can specify rated power and total harmonics distortion THD (refer to IEC 61000-2-4 paragraph 3.2.7) in p.u. of fundamental rms current. In this case harmonics spectrums are assumed to be as follows:

Harmonics: 3rd, 5th, 7th, 9th, 11th, 13th, 15th, 17th, 19th, 21st, 23rd and 25th

Harmonic “h” (from 3 to 25), p.u. value of fundamental rms, as follows:

\[
I_h = \frac{a I_1}{h} \quad [8] \\
I_1 = \left[ \frac{1}{1 + (\text{THD})^2} \right]^{0.5} \quad [9] \\
\alpha = \frac{\text{THD}}{0.463} \quad [10]
\]

And finally, eddy losses enhancement factor is found out by using the equation [6B].

In case customer specifies a certain K-Factor calculated according UL with exponent q=2, then, the harmonic spectrum shall be defined as above, and \( \alpha \) calculated as follows:

\[
\alpha = \left[ \frac{K - 1}{12 - 0.2145 \cdot K} \right]^{0.5} \quad [11] \\
K= \text{K-Factor}
\]

Therefore, the eddy losses enhancement factor should be found out by using the equation [6B].
As a summary of the previous explanation, the following table is starting the equivalence between the K-Factor according to the UL standards and the transformer over rating factor, the factor K.

**Table 1. Equivalence table between K-Factor (UL 1562) and over rating factor, Factor K (HD 538)**

<table>
<thead>
<tr>
<th>THD</th>
<th>0.136</th>
<th>0.194</th>
<th>0.241</th>
<th>0.281</th>
<th>0.350</th>
<th>0.413</th>
<th>0.472</th>
<th>0.501</th>
<th>0.529</th>
<th>0.585</th>
<th>0.641</th>
<th>0.727</th>
</tr>
</thead>
<tbody>
<tr>
<td>K-Factor UL</td>
<td>2.00</td>
<td>3.00</td>
<td>4.00</td>
<td>5.00</td>
<td>7.00</td>
<td>9.00</td>
<td>10.00</td>
<td>11.00</td>
<td>12.00</td>
<td>13.00</td>
<td>15.00</td>
<td>17.00</td>
</tr>
<tr>
<td>OVER RATING FACTOR K HD 538</td>
<td>1.02</td>
<td>1.04</td>
<td>1.06</td>
<td>1.08</td>
<td>1.12</td>
<td>1.15</td>
<td>1.17</td>
<td>1.19</td>
<td>1.20</td>
<td>1.22</td>
<td>1.25</td>
<td>1.28</td>
</tr>
<tr>
<td>Eddy = 0.15 RI2 loss</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$I_1$(pu $I_{rated}$)</td>
<td>0.991</td>
<td>0.982</td>
<td>0.972</td>
<td>0.963</td>
<td>0.944</td>
<td>0.924</td>
<td>0.914</td>
<td>0.904</td>
<td>0.894</td>
<td>0.884</td>
<td>0.863</td>
<td>0.842</td>
</tr>
<tr>
<td>$I_3$(pu $I_{rated}$)</td>
<td>0.097</td>
<td>0.137</td>
<td>0.168</td>
<td>0.194</td>
<td>0.238</td>
<td>0.275</td>
<td>0.291</td>
<td>0.307</td>
<td>0.322</td>
<td>0.336</td>
<td>0.363</td>
<td>0.388</td>
</tr>
<tr>
<td>$I_5$(pu $I_{rated}$)</td>
<td>0.058</td>
<td>0.082</td>
<td>0.101</td>
<td>0.117</td>
<td>0.143</td>
<td>0.165</td>
<td>0.175</td>
<td>0.184</td>
<td>0.193</td>
<td>0.202</td>
<td>0.216</td>
<td>0.233</td>
</tr>
<tr>
<td>$I_7$(pu $I_{rated}$)</td>
<td>0.042</td>
<td>0.059</td>
<td>0.072</td>
<td>0.083</td>
<td>0.102</td>
<td>0.118</td>
<td>0.125</td>
<td>0.132</td>
<td>0.138</td>
<td>0.144</td>
<td>0.156</td>
<td>0.166</td>
</tr>
<tr>
<td>$I_9$(pu $I_{rated}$)</td>
<td>0.032</td>
<td>0.046</td>
<td>0.056</td>
<td>0.065</td>
<td>0.079</td>
<td>0.092</td>
<td>0.097</td>
<td>0.102</td>
<td>0.107</td>
<td>0.112</td>
<td>0.121</td>
<td>0.129</td>
</tr>
<tr>
<td>$I_{11}$(pu $I_{rated}$)</td>
<td>0.026</td>
<td>0.037</td>
<td>0.046</td>
<td>0.053</td>
<td>0.065</td>
<td>0.075</td>
<td>0.079</td>
<td>0.084</td>
<td>0.088</td>
<td>0.092</td>
<td>0.099</td>
<td>0.106</td>
</tr>
<tr>
<td>$I_{13}$(pu $I_{rated}$)</td>
<td>0.022</td>
<td>0.032</td>
<td>0.039</td>
<td>0.045</td>
<td>0.055</td>
<td>0.063</td>
<td>0.067</td>
<td>0.071</td>
<td>0.074</td>
<td>0.078</td>
<td>0.084</td>
<td>0.090</td>
</tr>
<tr>
<td>$I_{15}$(pu $I_{rated}$)</td>
<td>0.019</td>
<td>0.027</td>
<td>0.034</td>
<td>0.039</td>
<td>0.048</td>
<td>0.055</td>
<td>0.058</td>
<td>0.061</td>
<td>0.064</td>
<td>0.067</td>
<td>0.073</td>
<td>0.078</td>
</tr>
<tr>
<td>$I_{17}$(pu $I_{rated}$)</td>
<td>0.017</td>
<td>0.024</td>
<td>0.030</td>
<td>0.034</td>
<td>0.042</td>
<td>0.048</td>
<td>0.051</td>
<td>0.054</td>
<td>0.057</td>
<td>0.059</td>
<td>0.064</td>
<td>0.069</td>
</tr>
<tr>
<td>$I_{19}$(pu $I_{rated}$)</td>
<td>0.015</td>
<td>0.022</td>
<td>0.027</td>
<td>0.031</td>
<td>0.038</td>
<td>0.043</td>
<td>0.046</td>
<td>0.048</td>
<td>0.051</td>
<td>0.053</td>
<td>0.057</td>
<td>0.061</td>
</tr>
<tr>
<td>$I_{21}$(pu $I_{rated}$)</td>
<td>0.014</td>
<td>0.020</td>
<td>0.024</td>
<td>0.028</td>
<td>0.034</td>
<td>0.039</td>
<td>0.042</td>
<td>0.044</td>
<td>0.046</td>
<td>0.048</td>
<td>0.052</td>
<td>0.055</td>
</tr>
<tr>
<td>$I_{23}$(pu $I_{rated}$)</td>
<td>0.013</td>
<td>0.018</td>
<td>0.022</td>
<td>0.025</td>
<td>0.031</td>
<td>0.036</td>
<td>0.038</td>
<td>0.040</td>
<td>0.042</td>
<td>0.044</td>
<td>0.047</td>
<td>0.051</td>
</tr>
<tr>
<td>$I_{25}$(pu $I_{rated}$)</td>
<td>0.012</td>
<td>0.016</td>
<td>0.020</td>
<td>0.023</td>
<td>0.029</td>
<td>0.033</td>
<td>0.035</td>
<td>0.037</td>
<td>0.039</td>
<td>0.040</td>
<td>0.044</td>
<td>0.047</td>
</tr>
</tbody>
</table>

Remark: this equivalence list is valid up to 3150kVA transformers.
2 Interpretation / example

Assuming a transformer is requested to supply 2000kVA (called as fundamental power according to the IEC 61378-1) while the harmonic content of the net is defined as per K-Factor 17, the equivalent power (which includes the harmonic effect) of such unit shall be equal to the fundamental power multiplied by the proper Factor K according to the table attached above.

That is:

Fundamental Power – 2MVA

Net harmonic content – K-17 => Factor K equals to 1.28

<table>
<thead>
<tr>
<th>THD</th>
<th>0.136</th>
<th>0.194</th>
<th>0.241</th>
<th>0.281</th>
<th>0.350</th>
<th>0.413</th>
<th>0.443</th>
<th>0.472</th>
<th>0.501</th>
<th>0.529</th>
<th>0.585</th>
<th>0.641</th>
<th>0.727</th>
</tr>
</thead>
<tbody>
<tr>
<td>K-Factor UL</td>
<td>2.00</td>
<td>3.00</td>
<td>4.00</td>
<td>5.00</td>
<td>7.00</td>
<td>9.00</td>
<td>10.00</td>
<td>11.00</td>
<td>12.00</td>
<td>13.00</td>
<td>15.00</td>
<td>17.00</td>
<td>20.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>OVER RATING</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FACTOR K HD 538</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Eddy = 0.15 Rl2 loss</td>
<td>1.02</td>
<td>1.04</td>
<td>1.06</td>
<td>1.08</td>
<td>1.12</td>
<td>1.15</td>
<td>1.17</td>
<td>1.19</td>
<td>1.20</td>
<td>1.22</td>
<td>1.25</td>
<td>1.28</td>
<td>1.33</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I1(pu Irated)</td>
<td>0.991</td>
<td>0.982</td>
<td>0.972</td>
<td>0.963</td>
<td>0.944</td>
<td>0.924</td>
<td>0.914</td>
<td>0.904</td>
<td>0.894</td>
<td>0.884</td>
<td>0.863</td>
<td>0.842</td>
<td>0.809</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I2(pu Irated)</td>
<td>0.097</td>
<td>0.137</td>
<td>0.168</td>
<td>0.194</td>
<td>0.238</td>
<td>0.275</td>
<td>0.291</td>
<td>0.307</td>
<td>0.322</td>
<td>0.336</td>
<td>0.363</td>
<td>0.388</td>
<td>0.423</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I5(pu Irated)</td>
<td>0.058</td>
<td>0.082</td>
<td>0.101</td>
<td>0.117</td>
<td>0.143</td>
<td>0.165</td>
<td>0.175</td>
<td>0.184</td>
<td>0.193</td>
<td>0.202</td>
<td>0.218</td>
<td>0.233</td>
<td>0.254</td>
</tr>
</tbody>
</table>

Equivalent Power – 2000 x 1.28 = 2560kVA

As a result of this, the transformer shall be physically sized as 2560kVA rated power and the temperature rise limit shall be guaranteed at such equivalent power as well.

Therefore, should the temperature rise test is requested, during the full load part of the test the current must be equal to the rated current multiplied by the proper Factor-K, i.e. assuming the no load voltage level on LV side is 417V (Dyn11), rated current will be 2000/(417*\sqrt{3}) = 2.77kA, however test must be run at 2.77x1.28= 3.54kA (corresponding to 2560kVA).