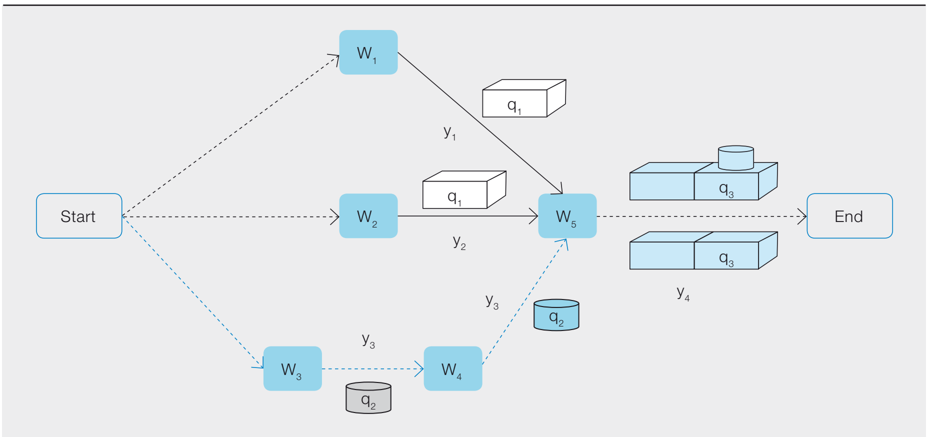
Reordering chaos

Applied mathematics improves products, industrial processes and operations

LUCA GHEZZI, ALDO SCIACCA – In a world of finite resources and an almost infinite number of binding constraints, mathematical modeling and simulation tools help optimize complex systems. Applied mathematics brings a rational outlook, precise problem definition and representation, qualitative and quantitative prediction capabilities, and the possibility to simulate how stochastic properties or unpredictable external forces affect system performance. Applied mathematics can yield significant savings in industrial processes and manufacturing as well as in the world of operations, such as in distributive logistics, production planning and sales force organization.

Title picture
For over 70 years, mathematics has been used to find optimal solutions to multivariable problems. The same techniques can also be used in factory settings to identify cost-effective production strategies.



1 A schematic representation of a simple production line, in the form of a directed graph

As frequently happens, wartime brings leaps in technology and science. Thus operations research (OR) and its main tool, mathematical programming (MP), blossomed during the dramatic years of World War II. Then, the idea was to use mathematics to solve literally life-saving problems such as where to locate the first few, and expensive, radar installations to spot and counter aerial offenses coming from the continent. A new method was needed to optimize a goal function that maximized the territory covered by the radar, bearing in mind physical, economic and integrality constraints – for it was not possible to locate one-quarter of a radar in Dover and the other three-quarters in Folkestone. The same techniques were used to determine the optimal size and composition of North Atlantic supply convoys.

The number of topics tackled by OR has become progressively larger – including combinatorial optimization problems that are constrained by equalities and inequalities and that function with continuous or discrete variables. Industrial processes, logistic networks, sales organization, scheduling and most other aspects of large organizations teem with examples of situations that can be optimized by an appropriate application of OR.

Mathematical programming

The mathematical response to the issue of combinatorial complexity is called mathematical programming, where a “program” is the problem of minimizing a goal function $f(x,z)$, subject to equality and inequality constraints $g(x,z)=0$, $h(x,z)\leq 0$, with real (continuous) and/or integer (discrete) variables. Robust approaches exist for those programs of a convex nature, for which the existence and uniqueness of a solution may be assumed in advance.

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The art of modeling consists of reducing complicated problems to as-simple-as-possible mathematical formulations that, nonetheless, retain the essence of the problem. For example, in a simple production plant, the goal function is typically the profit. This is a linear function of the product quantities sold and production costs, which latter, in turn, depend linearly on production quantities ➔ 1. The market requires a minimal quantity of products, which requires minimal quantities of subproducts, and so on down to raw materials, with each stage passing a minimal quantity constraint on to the next. At each work station in the process, the cumulative machine time required is constrained from above by the available machine time. Some stations require manpower to be drawn from finite capacity reservoirs (with due competence specialization constraints), which pose further inequality constraints. Intermediate buffers and warehouses can form dynamic item storage pools, to be depleted or increased, with lower bounds (safety margin against stockout), upper bounds (capacity) and additional cost contributions.

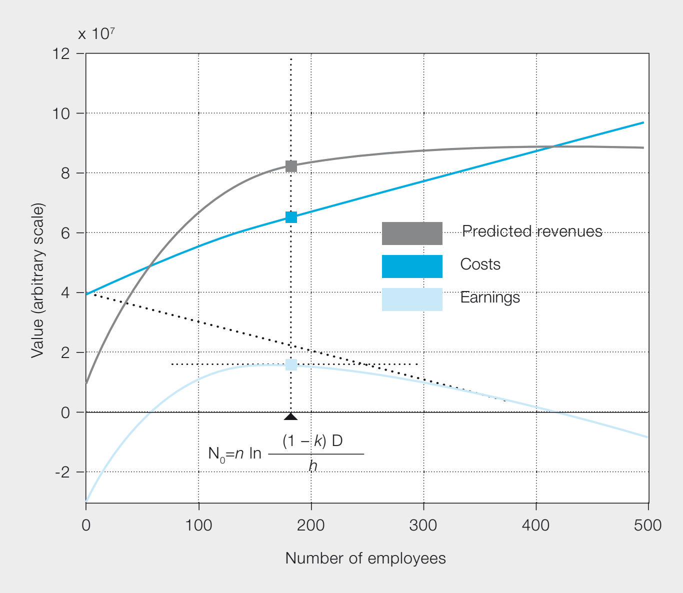
All this can be made into a convex program and MP will deliver an optimal management strategy. A large collection of building blocks, with thousands of variables and linear constraints, allows a complete production line, or even a factory, to be modeled, inclusive of outsourced stages. Indeed, this very exercise was performed for ABB’s electric motor plant in Vittuone, Italy.

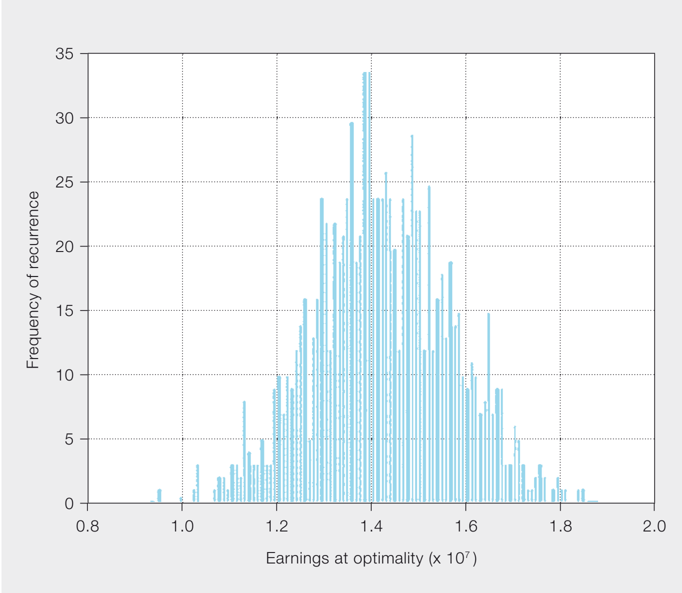
Graph theory

As can be seen in the example above, a graph offers a clear, ordered, intuitive and visual representation of the problem to be solved. Indeed, graph theory is a major OR tool, for solution purposes as well as for representation.

From the graph theory standpoint, all arbitrary systems represented by nodes and connections may be treated the same. Therefore, a distribution logistics

2 A simple approach yields results





2a Predicted revenues (grey), costs (blue) and earnings (light blue) vs. sales force, with optimal sizing

2b Propagation of variance over earnings in a sample, illustrative case

network or a supply net can be graphically modeled in the same way as a production plant. Customers to be served, transit points, logistic hubs, warehouses and production equipment are the graph nodes, while admissible routes are the connecting lines. Capacity constraints affect transport, handling operations and production. Market demands pose lower bounds for goods dispatched (others go to warehouse inventory) and everything is given a cost.

Many canonically formulated MP problems for network optimization equate to classical graph theory problems, for which simple but powerful theorems yield exact solutions – directly and without too much number crunching.

Part of the ABB logistic network in Italy has been simulated in this way in order to find an optimal management strategy.

Heuristics

Often, computing an exact solution takes too long. Further, data is often intrinsically uncertain and mutable, and a safety margin is usually needed in any case. Therefore, a suboptimal solution may often be a more reasonable option.

Based on different approaches, such as the application of sequences of improvement rules or the imitation of physical, human or biological system evolution, heuristics are discovering methods that lead to a solution that is not necessarily the optimum but is frequently acceptably close to it. Sometimes, the clue for a proficient heuristic method can come from pure mathematics.

For instance, by following, in a heuristic fashion, the very same steps as the celebrated Euclidean algorithm for the greatest common divisor of integers, it is possible to sequence a collection of different items belonging to some given families, such as products to manufacture in a production plan, to try to maximize their scattering (interestingly, here more chaos is sought, rather than less).

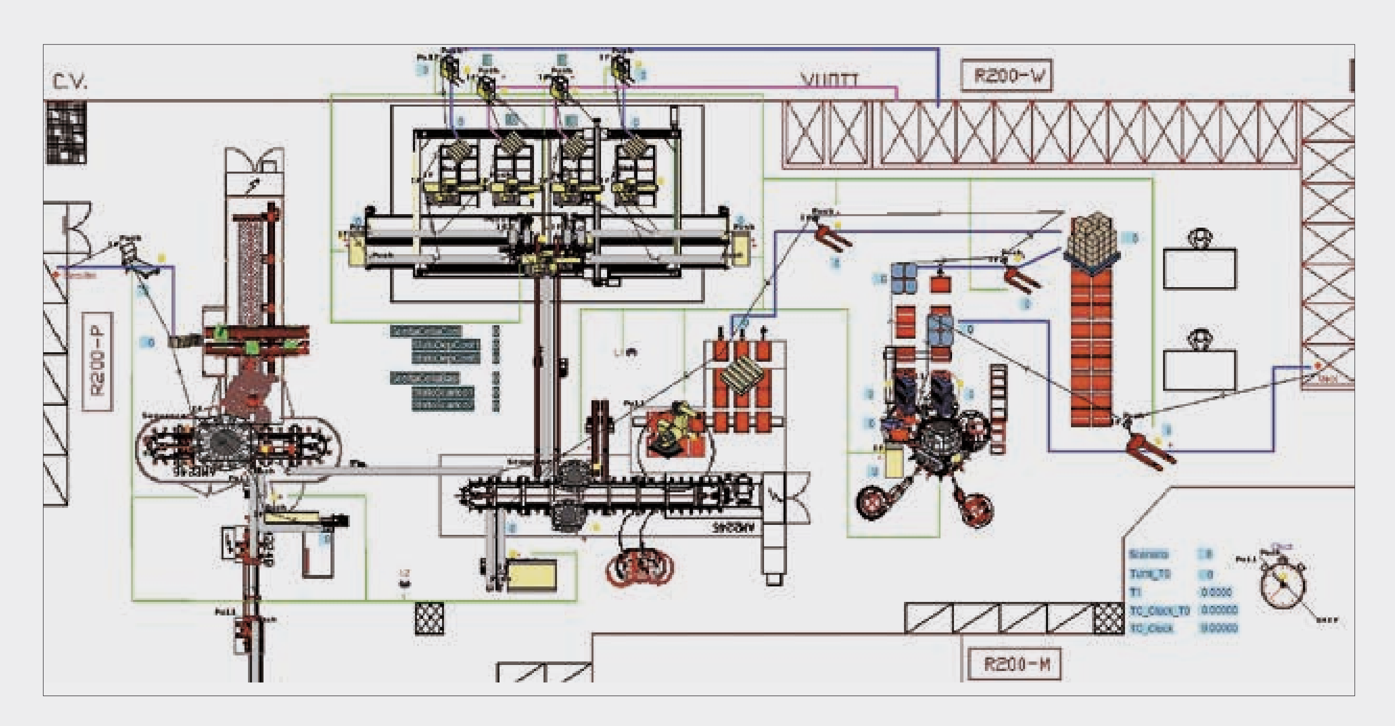
This method, along with other, adaptive, production-mix-based, heuristic approaches and empirical recipes deduced from field best practices, is now a component of the short-term (one-week) line scheduler in use at the ABB vacuum circuit breaker plant in Dalmine, Italy. The blending of different tools into a flexible and reactive system, easily interacting with human intelligence, is itself a heuristic super-tool, in response to complexity and chaos.

Exact analytic modeling

Problem simplification is always a good first approach. A feeling for variable sensitivity and order-of-magnitude effects also helps problem formulation. This is exactly what is required, for example, when simulating sales figures – a saturating revenue curve, modeling both

Often, computing an exact solution takes too long so, in many cases, a suboptimal solution may be a more reasonable option.

3 Discrete event simulation of the "end-machine" portion of F200 residual current device assembly line in ABB's Santa Palomba plant in Italy



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the virgin and the mature market regimes, may be described by a differential equation with some parameters that can be derived from available historical market data. Coupled with a revenue-dependent cost curve, a differentiable analytical model can be generated that is simple enough to solve in closed form – allowing earnings and their maximal values to be identified ➔ 2.

Stochastic systems

Many processes have uncertainty in their input data, so these are often expressed as a probability density function (PDF) that describes, for each input, the probability of each of their admissible values occurring. Naturally, the process output data is then nondeterministic. Various methods are available to describe this unpredictability.

The celebrated Monte Carlo (MC) method constructs the output PDF by running many (often over a million) randomly chosen inputs through a black box deterministic simulator. The asymptotic convergence rate is independent of the number of input data items (this does not mean that with few or many random variables the convergence time is the same). MC is simple to understand and cheap to implement, but is often too slow.

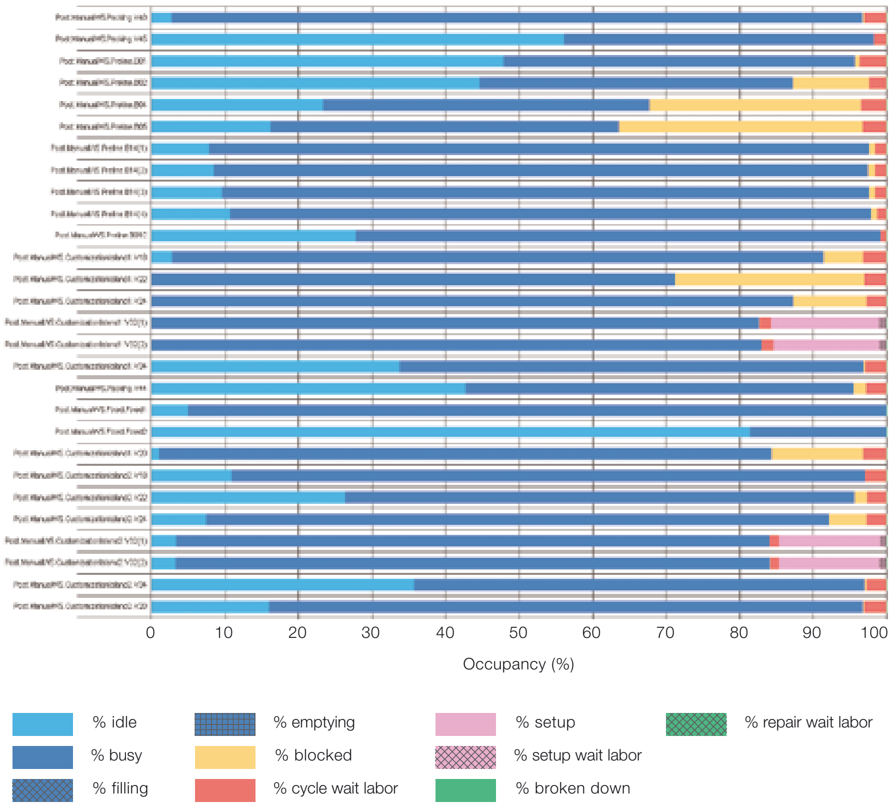
A recent alternative is the so-called polynomial chaos (PC) method. The basic idea is to expand the output PDF into a truncated series of known basic functions; for instance, orthogonal polynomials (hence the name). Polynomial orthogonality allows rapid determination of the expansion coefficients, typically with only very few runs of the deterministic simulator.

Discrete event simulation

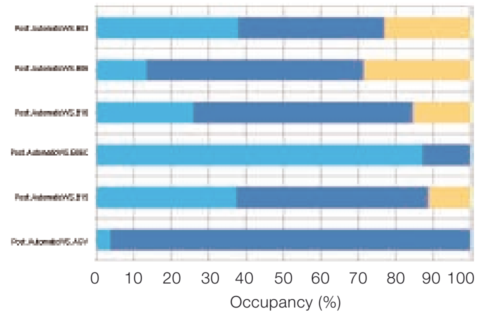
High-level, strategic, quantitative methods, such as MP, help, but are not sufficient, to accurately mirror real-life situations. Late part delivery, machine breakdown and maintenance, manpower scheduling and the other complicating factors that dog typical production operations also have to be taken into account.

A solution is discrete event simulation. This involves a virtual replica of the factory (or logistic network, warehouse, etc.) that reflects items passing through the different production stations and the effects on the process of deterministic and stochastic interference. Commercial tools can be used to reproduce the real system to the required level of detail and to run different scenarios to determine the optimal manpower distribution, machine allocation, scheduling strategy and so on.

4 Occupancy after a discrete event simulation. Color codes indicate busy, blocked upstream or downstream, waiting for labor or repair, etc.



4a Machine occupancy



4b Manpower occupancy

Several ABB factories have been analyzed with discrete event simulations. These yield visual and intuitive representations, like CAD views of the layout that are animated with workers, parts and products moving from machine to machine ➔ 3. One of the main results is a set of tables showing the actual amount of time spent processing by workers and machines, or idle time caused by, eg, upstream or downstream bottlenecks or repair demands ➔ 4. This information is fundamental to appropriate resource allocation.

A continuous effort

In some cases, a one-off simulation suffices. In others, the simulation may become an integrated tool, to be used on an ongoing basis. Either way, if a constant, empirical validation is absent then effective modeling and simulation is hardly possible.

The key prerequisites for a successful implementation of the techniques discussed here are a firm commitment together with a well-structured approach to data collection and management. The latter implies an investment that goes beyond the implementation of a given IT solution and whose return is tangible also in absence of simulations.

Simulations quantitatively evaluate cost-saving or empowerment actions before they are taken and enable better-run operations both in the short-, medium- and long-term. So, a thorough and skillful application of MP techniques can make significant improvements to a company's bottom line.

Luca Ghezzi
Aldo Sciacca
ABB LPED
Milan, Italy
luca.ghezzi@it.abb.com
aldo.sciacca@it.abb.com