THE BROWN BOVERI REVIEW

To-day high-grade welding goes far to replace bolts and flanges.
Governing valves and housing of a high-pressure primary turbine with welded on nozzle-valve casings and admission steam pipes.

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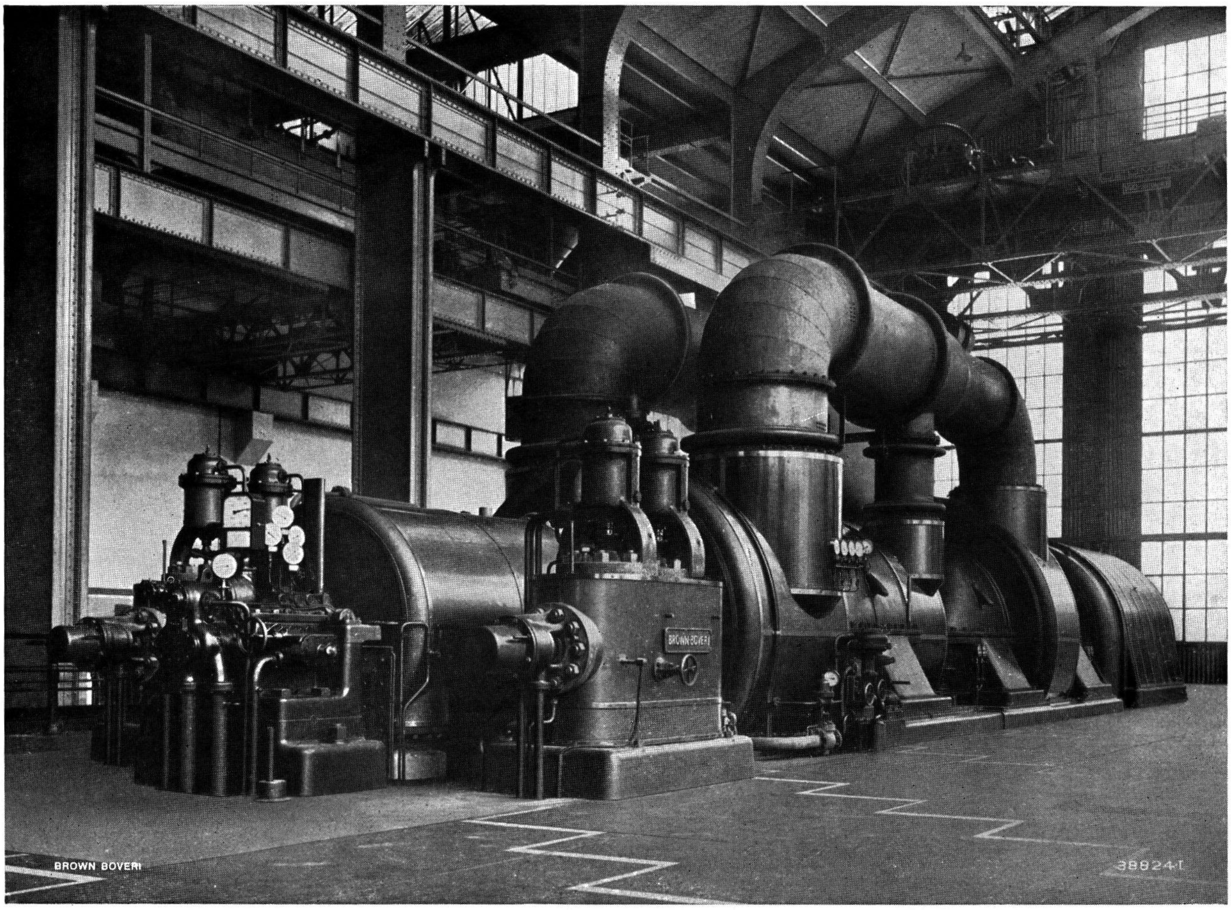

Steam turbine 50,000 kW, 3000 r. p. m.

40 YEARS OF
BROWN BOVERI STEAM TURBINES

5000 PLANTS

20 MILLION KILOWATTS
(EXCLUDING MARINE PLANTS)

THE BROWN BOVERI REVIEW

THE HOUSE JOURNAL OF BROWN, BOVERI & COMPANY, LIMITED, BADEN (SWITZERLAND)

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THE CREATION OF THE BROWN BOVERI STEAM TURBINE.

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A summary is given here of the development of the Brown Boveri steam turbine. After the first units had been built, the design of which followed very closely that of the Parsons model, turbines were designed which, while adhering to the fundamental Parsons principle of reaction blading, were increasingly distinguished by unique features of their own. After the so-termed "combined turbine" and the "disc-type turbine", which are both characterized by having impulse blading in the high-pressure part, the tendency is to return to reaction blading with a large number of stages divided up between several cylinders, especially in the case of units for big outputs; finally we also find the single-cylinder turbine coming into more frequent use again, new manufacturing processes such as welding being now applied in the construction of these units.

IN the year 1900, the first Parsons turbine on the Continent was erected in Elberfeld. It came from England and aroused well deserved interest. In the same year we decided to acquire the licence rights for building similar turbines. It will be understandable that in the first few years we adhered closely to our Parsons forerunner, adopting as well the Parsons methods of calculation and making use of the experience already acquired at that date by Parsons. Not only were the typical features of the Parsons turbine, such as reaction blading, rotor drum, balancing piston and labyrinth sealing wonderfully well adapted to their various purposes but details such as bearings, couplings, etc. reflected the exceptional technical ability and wide experience of their inventor.

As time passed, and as we gained experience of our own, we added features which differentiated our turbines from the original Parsons model. This increasing differentiation was due not only to the different requirements and conditions pertaining to the British and to the Swiss markets but was also due to different methods of tackling and solving new problems, which characterized British and Swiss engineering.

The design of the bearings was modified at an early date; we also did away with the cast-in channels laid along the casing for balancing the thrust, in order to simplify the cylinder, and developed a new type of regulator in 1905.

At that time, the Parsons turbine had one single admission valve, controlled by steam the pressure of which was adjusted from the regulator by means of a relay pin. We replaced steam by oil for actuating the main valve; additional valves already introduced by Parsons remained operated by hand or steam piston. In 1908, however, we connected these valves as well to the oil governing system and thus created our rodless oil-pressure governing system. The latter has found a wide field of application and it is certainly one of our most significant contributions to the technology of governing. The oil-pressure governing system has been in use up till to-day, its excellent fundamental features remaining unchanged.[1]

Let us return, now, to the first turbines we built. The very first order booked was in 1902, for the Frankfort Electricity Works, for a 2600-kW unit at 1360 r.p.m.,[2] that is for a turbine of more than double the output of the only other one on the Continent, the famous Elberfeld machine. A really big turbine, for these days, was the one delivered, two years after the Frankfort turbine, to the Rheinisch-Westfälische Electricity Works in Essen, delivering 5000 kW at 1000 r. p. m. (Fig. 1). In 1907 we built a two-cylinder turbine 7500 kW, 750 r. p. m. for Buenos Ayres. This machine shows several original features of our

[1] The Brown Boveri Review, 1915, page 59.

[2] The Brown Boveri Review, 1935, page 38.

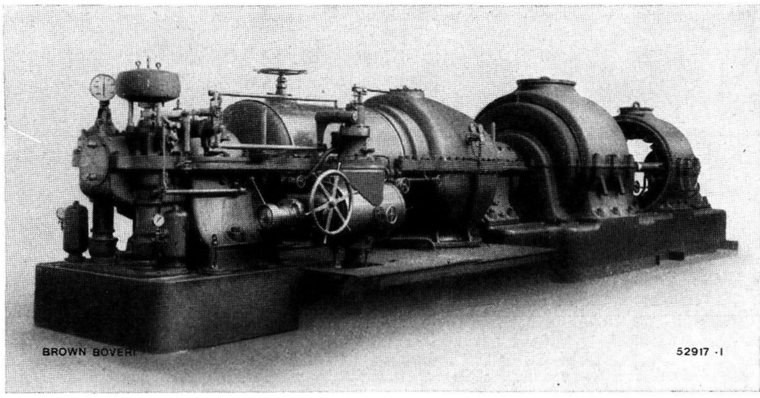


Fig. 1. — Brown Boveri Parsons turbine 5000 kW at 1000 r. p. m. for the Rheinisch-Westfälische Elektrizitätswerk, Essen.
Built in 1904.

own, but more from the point of view of inner design (Fig. 2) than from that of outward appearance (Fig. 3). At that period, steam pressures were generally below 10 kg/cm^2. As the steam pressures and, above all, steam temperatures worked with, increased, we were faced with new problems. The cast-iron cylinders had to be replaced by cast-steel ones. It became difficult to maintain the blading efficiency which had been satisfactory up till then, because, as the pressure rose, and the adiabatic heat drops became bigger, it was difficult to reconcile height of blade, mean diameter of blading and number of stages. Thus, we find the first suggestions put forward to build a combined turbine; the high-pressure part was designed as an impulse wheel with two or three rows of blades, the low pressure part with Parsons blading beginning after the steam had sufficiently expanded to allow of its having sufficient volume for blading of a favourable height. As, however, the impulse wheel was very much narrower than would be a reaction part built to deal with the same adiabatic heat drop, it became possible to put more stages in the remaining reaction part, thanks to the space saved in the high-pressure part; in this way what was lost in efficiency in the impulse wheel was recuperated in the efficiency of the reaction blading.

A milestone in turbine manufacture progress was the unit built in Mannheim in 1914, before the out-break of the World War, for the E. W. Mark in Westphalia, with an output of 40,000 H. P. at 1000 r. p. m. (Fig. 4). Here, the oil-pressure governing system is used, the main throttle valve and additional valves being characterized by the feature that the nozzle valves opened suddenly and fully one after another. This arrangement was given up, later. Since 1916,

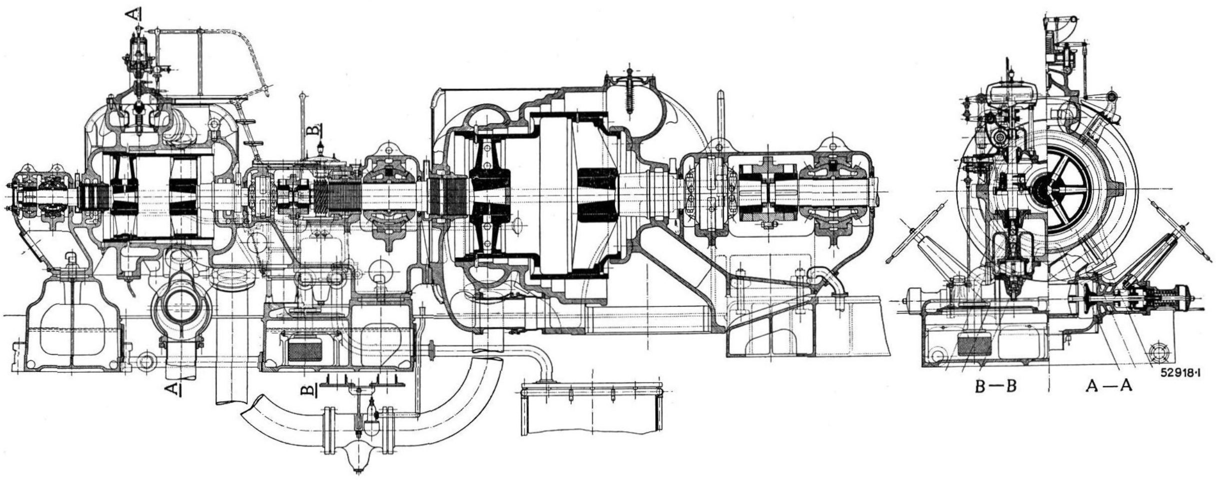


Fig. 2. — Longitudinal and cross section of the governing system and inlet valve parts of the two-cylinder turbine for Buenos Ayres, 7500 kW, 750 r. p. m., as an example of turbine design at that period.
Built in 1907.

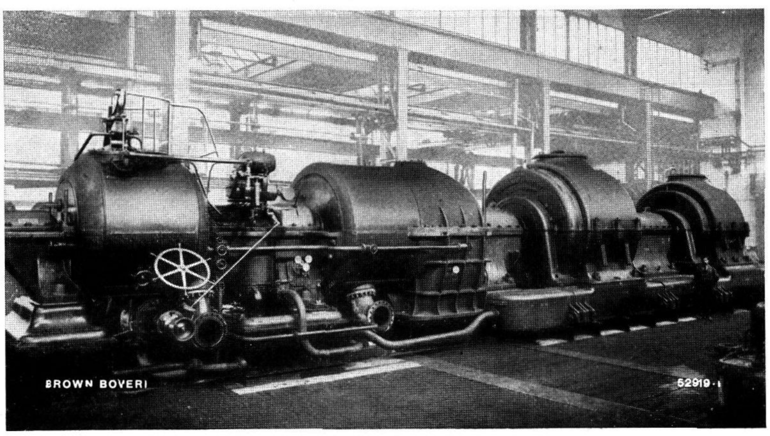


Fig. 3. — Two-cylinder turbine, Buenos Ayres (see Fig. 2).

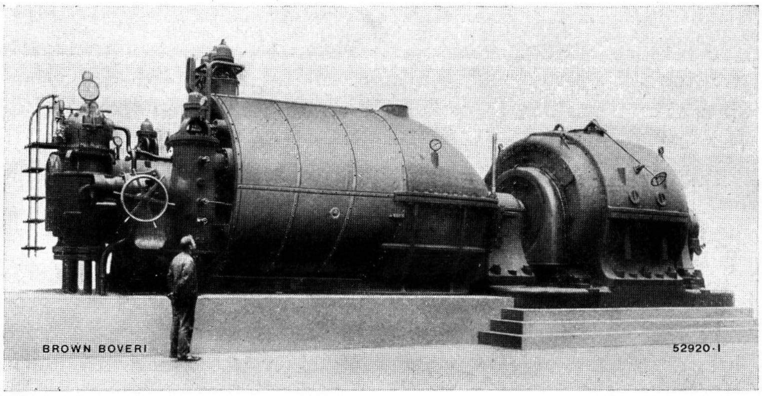


Fig. 4. — Design of 1914. Biggest single-cylinder turbine in the world for 40,000 H. P. at 1000 r. p. m. for the Mark power house of the Rheinisch-Westfälische Elektrizitätswerk.

the steam is carried straight to the nozzle valves and the latter are made to open gradually one after the other, each nozzle valve being connected to its own set of nozzles. In this way, the steam pressure is utilized at all load stages with a minimum of throttling and heat-drop losses[1]. The constantly increasing power demands made on the electricity works resulted in turbine units of increasing size being called for. Apart from the increase in output, it appeared advantageous to increase the speed of the sets, in order to make them both lighter and smaller; or as the highest speed of the generator was limited to 3000 r. p. m., the problem was rather to get the maximum of output from the turbine at that speed.

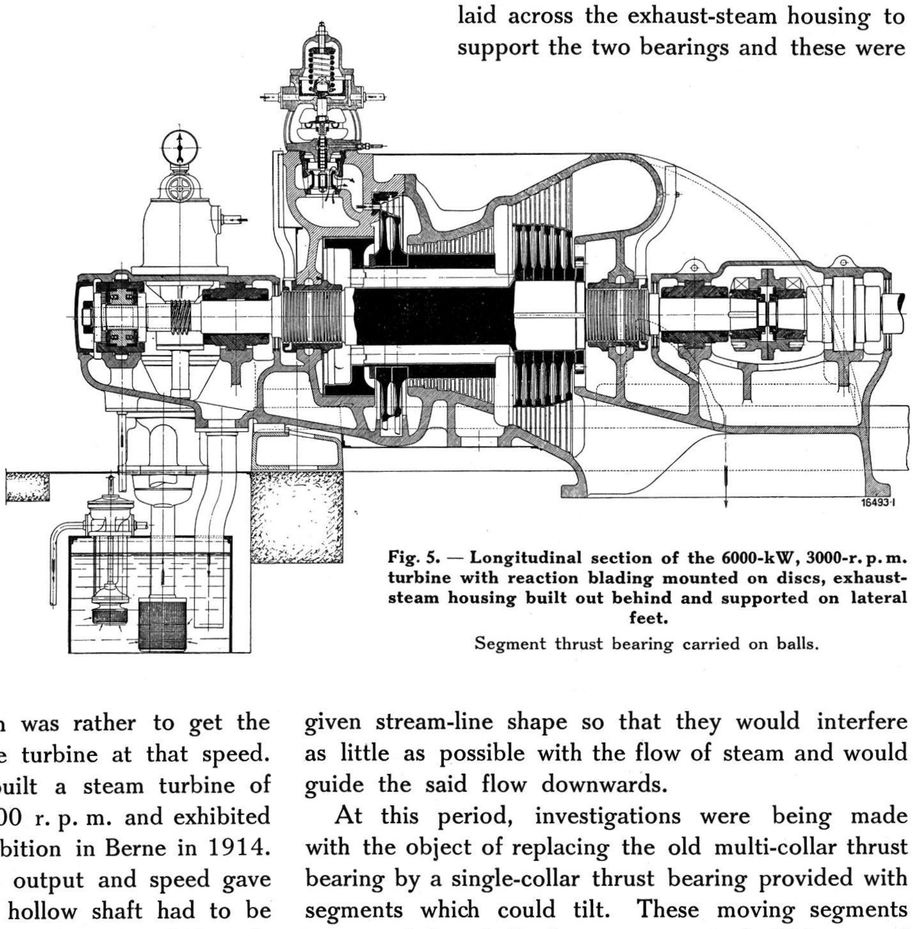


Fig. 5. — Longitudinal section of the 6000-kW, 3000-r. p. m. turbine with reaction blading mounted on discs, exhaust-steam housing built out behind and supported on lateral feet.

Segment thrust bearing carried on balls.

For the first time, we built a steam turbine of 6000 kW for a speed of 3000 r. p. m. and exhibited it at the Swiss National Exhibition in Berne in 1914. The simultaneous increase in output and speed gave rise to new problems. The hollow shaft had to be given up and replaced by a massive one which only carried the medium-pressure reaction blading; the low-pressure part was composed of discs which were designed to transmit to the hub, with an ample margin of safety, the stresses imposed by the long low-pressure blades. As the blading at the exhaust end got bigger, it appeared suitable to redesign the exhaust-steam housing. Owing to the design in general use in which the support of the housing at the low-pressure end was underneath the rear turbine bearing and first generator bearing, it was necessary to restrict the section of the exhaust steam housing over the low-pressure blading and this led to unfavourable steam flow conditions and big exhaust losses. Two feet are, now, provided on either side to carry the exhaust-steam chamber. The neighbouring turbine and generator bearings are entirely embedded in the exhaust housing so that the whole exhaust can extend towards the rear as far as is allowed by the foundations of the generator without extending the length of the whole set. Two ribs were laid across the exhaust-steam housing to support the two bearings and these were given stream-line shape so that they would interfere as little as possible with the flow of steam and would guide the said flow downwards.

At this period, investigations were being made with the object of replacing the old multi-collar thrust bearing by a single-collar thrust bearing provided with segments which could tilt. These moving segments were carried on balls the arrangement of which assured equal loading of all the segments.[2]

The new rotor construction of the turbine allowed of attaining an output of 12,000 kW at 3000 r.p.m. and one of 30,000 kW at 1500 r.p.m. also under good vacuum and satisfactory conditions as regards the exhaust steam end. To increase the output still further, which was the aim both of the turbine and

[1] The Brown Boveri Review, 1924, page 180; 1932, page 155.

[2] The Brown Boveri Review, 1917, pages 3, 35, 58 and 80.

generator designers, meant a reduction of the hub diameter of the low-pressure discs in order that the stresses due to the longer blades would be withstood. Thus, in 1920, we brought out the Brown Boveri disc turbine (Fig. 6). This turbine comprises a shaft of relatively small diameter which was made rigid in the first units built and flexible in subsequent ones; this shaft carries a row of impulse and reaction-blading discs. At a superficial glance, this turbine ressembles an impulse turbine, however, it is only an impulse turbine in as far as the first discs are concerned, which is shown by the intermediate diaphragms between the discs in question. The low-pressure part has reaction blading and here we find the application of our gap-bridging device used for the first time.[1] Instead of having the radial gap characteristic of reaction turbines, we find here a shrouding which limits the gap. In order to prevent the steam flowing over the shrouding, the radial limitation of the blade channel is graded in steps by means of intermediate pieces and shrouding in such a way that the pressure difference between the steam flow and the space surrounding it is bridged over by injector and ejector action. This gap bridging had the great advantage that the runner blades could be mounted on discs the rims of which did not have to be widened for the purpose of limiting the radial gap but could be designed just wide enough to withstand the centrifugal forces exercised by the blades (Figs. 5 and 6). The corresponding lightening of the rim allowed of having bigger blading diameters and blade lengths without overstressing the hub. Thanks to the elimination of the widening of the disc rim just mentioned, it became possible to widen the guide blades and the hubs so there was nothing to prevent enlarging the runner blades and the output of the turbine. This blading principle was adopted later for the low-pressure blading of two- and three-cylinder turbines and is still in use to-day.

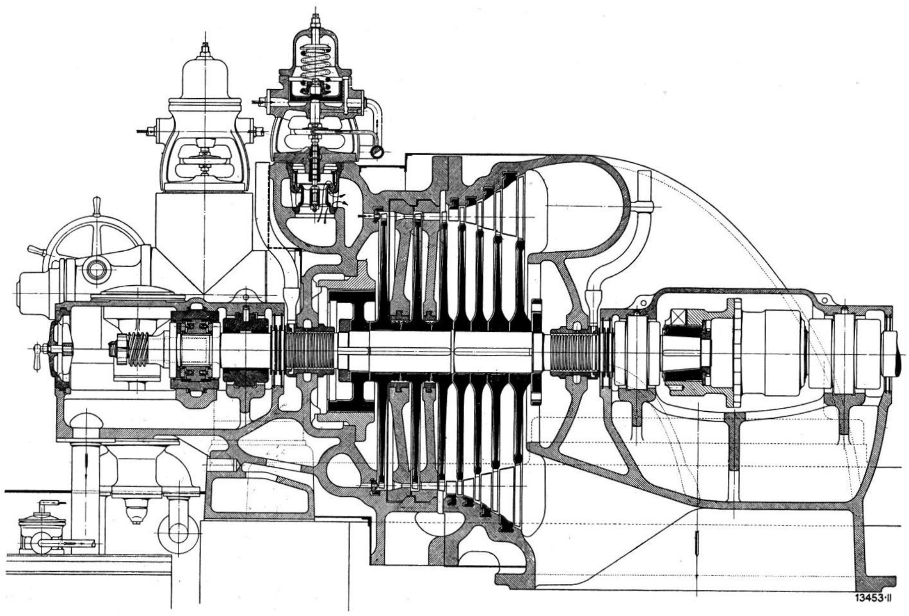


Fig. 6. — Longitudinal section of the disk-type turbine built for 12,000 kW at 3000 r. p. m.
3 impulse wheels and 2 intermediate diaphragms, 4 reaction wheels with Brown Boveri gap bridging device, shrouding on guide and runner blades. — Built in 1920.

The disc-type turbines were found to be very competitive despite the fact that their efficiency was not very high and that they were expensive to build. They were yet another proof that short turbines of big diameter are more expensive than long ones of small diameter. It is true that the actual purchasing price was not the chief factor, the running costs being the deciding one, at that period. It must be remembered that for a long time after the War a ton of coal cost about Fr. 200.—. If, now, it was thought worth while to pay a higher price for a turbine in order to get a couple of per cent higher efficiency from it, it is understandable that it was considered still wiser to invest the extra outlay in such turbine designs as promised far higher efficiencies than anything of the kind then on the market.

Efforts to improve the overall efficiency of the plant appreciably were in two directions. One was to increase both steam pressure and steam temperature which was generally carried out in conjunction with new operating processes such, for example, as bleeding for preheating the condensate and intermediate superheating of the steam[2]. The other direction was towards increasing the number of stages, improving steam flow conditions, careful avoidance of all losses and the like. In order to allow of introducing very high steam pressures it was found to be advantageous to leave existing steam power stations as they were, and, in the first period of this development, to carry out the transformation to very high pressure for the basic load on the station alone, the high pressure steam in question being expanded down in special turbines which then delivered the steam as back-pressure steam to the low-pressure piping already

[1] The Brown Boveri Review, 1925, page 200.

[2] The Brown Boveri Review, 1924, pages 23 and 54.

existing. To this purpose, we created the primary turbine designed for low outputs or very high pressures as a one-cylinder or a multiple-cylinder impulse turbine[1], and for bigger outputs as a combined turbine, that is with impulse or Curtis wheel and reaction blading after it.

The first primary turbine of 1650 kW for a pressure of 56 kg/cm^2 abs was built for Langerbrugge in Belgium[2] and was the first very high pressure plant in the world. Soon afterwards Brown Boveri Mannheim built a much bigger unit for the Mannheim Electricity Works and then we built a 4000-kW unit for the highest pressure ever used up till that day, namely 200 kg/cm^2 abs, which was again for the Langerbrugge plant[3].

When once technical circles had gained entire confidence in high-pressure, and it was considered a superfluous precaution to insist on two different steam systems in the same station, turbines for very high pressure were developed which used the steam down to vacuum, in one machine. Thus, we had the opportunity of delivering a whole series of big high-pressure and very high-pressure condensing turbines of which we will only mention one (Fig. 7). This is a 36,000-kW unit, for 120 kg/cm^2 abs at 500^0 C for the Caroline Pit in Witkowitz, Moravia. This turbine has delivered far over 800 million kWh in over 70,000 running hours, since 1932[4].

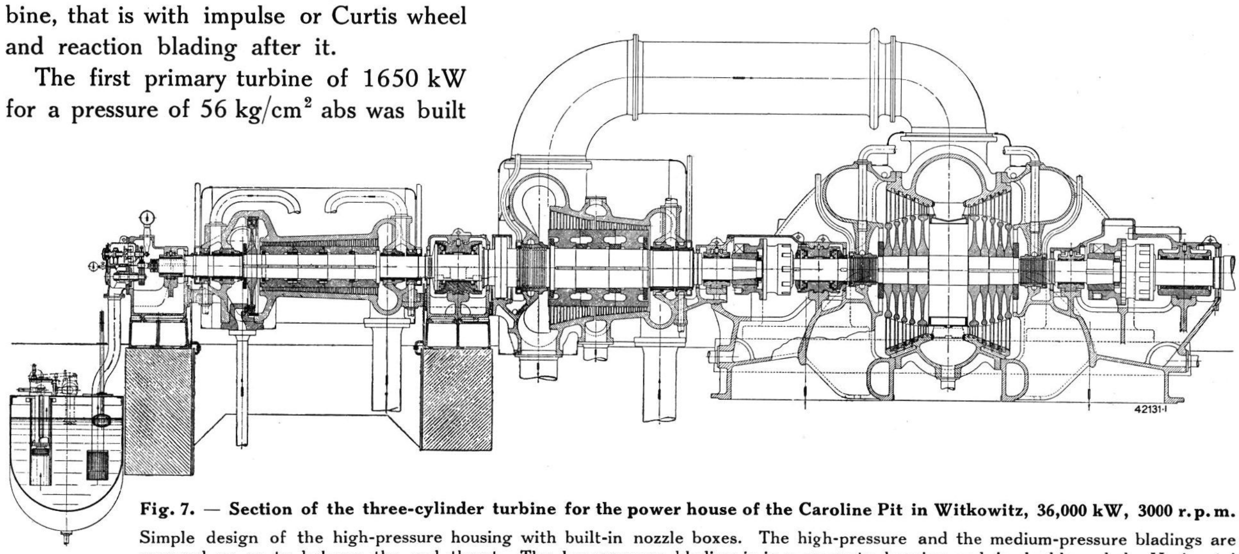


Fig. 7. — Section of the three-cylinder turbine for the power house of the Caroline Pit in Witkowitz, 36,000 kW, 3000 r.p.m.
Simple design of the high-pressure housing with built-in nozzle boxes. The high-pressure and the medium-pressure bladings are opposed so as to balance the end thrust. The low-pressure blading is in a separate housing and is double-ended. Horizontal governing and gear-type oil pump. Thrust and carrying bearing combined. — Built in 1930/31.

The other important line of development was, as already mentioned, the increase in the number of stages and, combined therewith, the subdivision of the heat drop among several cylinders[5]. In this way, every section of blading can be given the most advantageous blading

Fig. 8. — Power house of the Caroline Pit. The Brown Boveri turbo-set in foreground (see Fig. 7).
Valve housing separate. U-shaped connection pipes between valve housing and nozzle boxes.

[1] The Brown Boveri Review, pages 172/173; 1926, page 30.
[2] The Brown Boveri Review, 1928, page 32.
[3] The Brown Boveri Review, 1930, page 41.
[4] The Brown Boveri Review, 1931, page 45; 1934, page 92.
[5] The Brown Boveri Review, 1924, page 175; 1925, page 199.

diameter and the temperature drop per cylinder kept moderate; the thrust is easy to balance so that balancing pistons with the losses inherent thereto can be dispensed with. In the case of big quantities of steam, the low pressure part is made double-ended, the turbine being then given three cylinders, so that the axial thrust of the rotors of the first two cylinders equalize one another and the rotor of the third cylinder has no thrust thanks to its being double-ended. For smaller amounts of steam for which the blading of a single low-pressure part suffices, two cylinders are used, the pressure of the high-pressure rotor being balanced by that of the low-pressure rotor[1].

At the same period, about 1925, we developed a new reaction-blade profile which allowed an immediate improvement of several per cent in the blading efficiency.

With our three-cylinder turbine, it can be asserted that the steam turbine attained a stage of development which can hardly be surpassed as regards efficiency, mechanical properties and beauty of line (Fig. 8). This type of turbine was much appreciated and more than a hundred of them delivering a total of over $2\,{}^{1}/_{2}$ million kW were delivered in the course of a few years, only.

In the case of turbine outputs exceeding 35,000 kW and working with high vacuums, the speed adopted was 1500 r.p.m. The biggest unit of this class is the turbine in Zschornewitz near Berlin, delivering 85,000 kW. Another 1500-r.p.m. turbine is in the Gennevilliers plant in Paris; it delivers 50,000 kW. At the request of the purchaser, the machine is equipped with four condensers, two above and two below the engine-room flooring. In the case of the turbine delivered to the St. Denis Station, Paris[2], the speed chosen was 3000 r.p.m. despite the big output of 50,000 kW; there are three low-pressure bladings working in parallel to handle the very big quantity of low-pressure steam (Fig. 9).

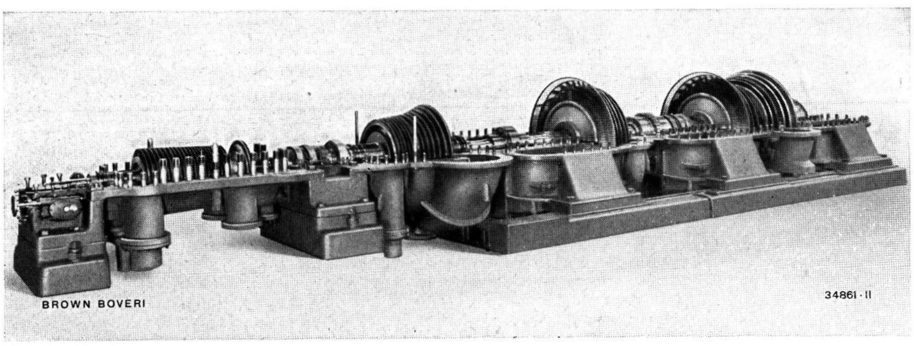


Fig. 9. — 40/50,000 kW turbine, 3000-r. p. m. St. Denis Power Station, Paris.

Lower parts of cylinders with rotors in position. The high-pressure blading and the one-ended low-pressure blading are opposed to the medium-pressure blading to balance the thrust. — Built in 1930/31.

The 165,000-kW turbine we delivered to the Hellgate Power Station in New York forms a milestone in the development of multi-cylinder turbines[3]. The problem put to us was to get the biggest possible output into the very restricted space available in this old power house. An exciting struggle began between the various turbine builders each trying to excel the others by a bigger output and ingenious design. Finally, we got the output up to 165,000 kW and secured the order. We had chosen a two-shaft, two-cylinder arrangement (Fig. 10). The steam inlet in the high-pressure cylinder is on the turbine coupling end and the steam outlet from the high-pressure cylinder is opposite the inlet branch to the double-ended low-pressure cylinder. There was just room for the massive valve housing in front of the high-pressure cylinder, that is beside the front part of the low-pressure housing. The outputs of both halves of the unit are about equal, the high-pressure part runs at 1800 r. p. m. and the low-pressure part at 1200 r. p. m., a speed which could not be exceeded because of the enormous low-pressure discs of 3 m diameter and 5 m diameter up to the edges of the blades (Fig. 11). The dimensions of this turbine exceeding anything built before, naturally set the designers and builders a whole series of new problems to solve. This was done and the first load test on site can be said to have shown no defect. Unfortunately, the subsequent servicing of this machine was confided to an American firm, on servicing and political grounds.

For nearly a decade, the multi-cylinder turbine dominated the market for high-output turbines, but the single-cylinder unit then began again to attract the attention of engineers; we may add that we had not neglected throughout this period to develop this type of machine, in the possibilities of which we had confidence. The price of coal had now dropped and the requirement of the very lowest steam consumption as compared to first cost of the plant had ceased to be of primordial importance. Cheaper turbine designs began to be demanded although, even under the new conditions, it remained an open question

[1] The Brown Boveri Review, 1926, page 30.

[2] The Brown Boveri Review, 1930, page 37; 1932, page 31; 1941, page 342.

[3] The Brown Boveri Review, 1927, pages 30/40; 1928, page 37; 1929, page 39.

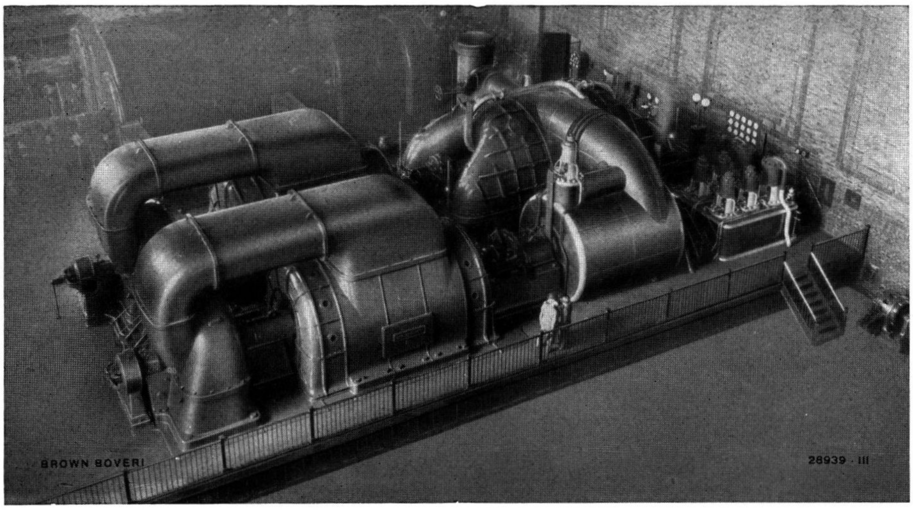


Fig. 10. — Hellgate power station with the Brown Boveri turbo-set in the foreground. Total output 165,000 kW at 1800 and 1200 r. p. m. respectively.

Built in 1926/28.

The short high-pressure part and the steam inlet placed on the coupling side allow of favourable layout of the pipe carrying steam from the high to the low-pressure turbine and allow of lodging the valve housing in the remaining free corner of the space available.

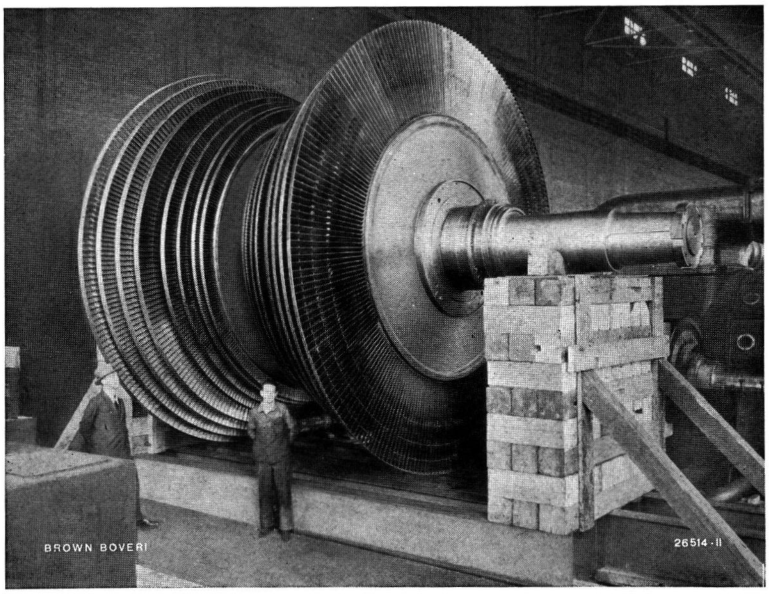


Fig. 11. — Low-pressure rotor of Hellgate turbine completely bladed.

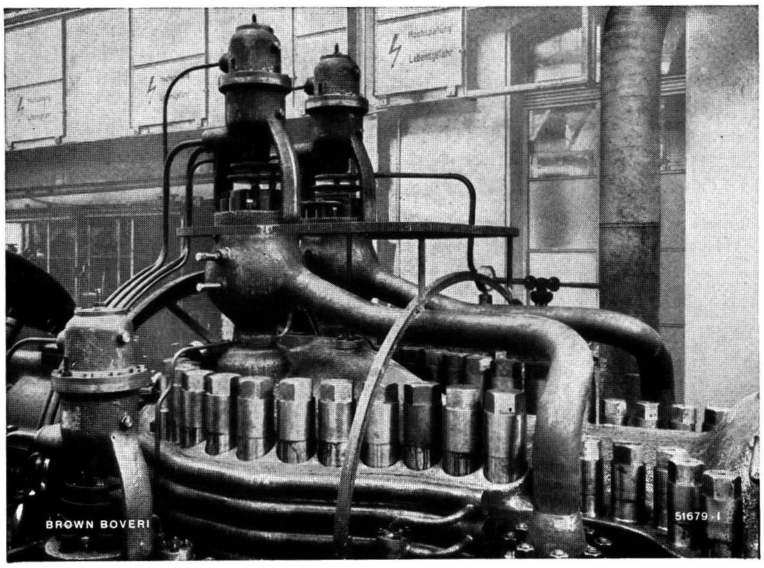


Fig. 12. — High-pressure primary turbine 15,000 kW, 3000 r. p. m. for 120 kg/cm², 500° C, on the test bed.

Much welding work incorporated. Valve housings are welded on to the collars of the nozzle boxes. The latter are also welded into the high-pressure housing. All steam-carrying pipes are welded to the housing. External heating of the flanges, to prevent too great temperature differences and consequent heat stresses, by means of welded on semi-circular ducts.

Built in 1939.

whether the "peak-load turbines", put forward by some firms to complete existing plants, could be considered as an advisable solution. We were always of the opinion that a new turbine which is also called

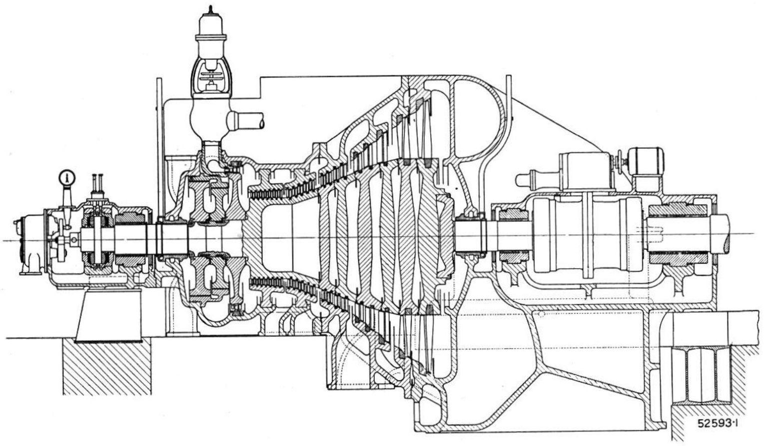


Fig. 13. — Section of the combined turbine 25,000 kW at 3000 r. p. m. with one-ended exhaust.
Rotor composed of a bell-shaped front part and 6 reaction discs held together at their periphery by welding. The impulse wheel and balancing piston are secured by lip welding to the front part of the shaft. Nozzle boxes welded in with valve casings welded on over them. Built in 1940.

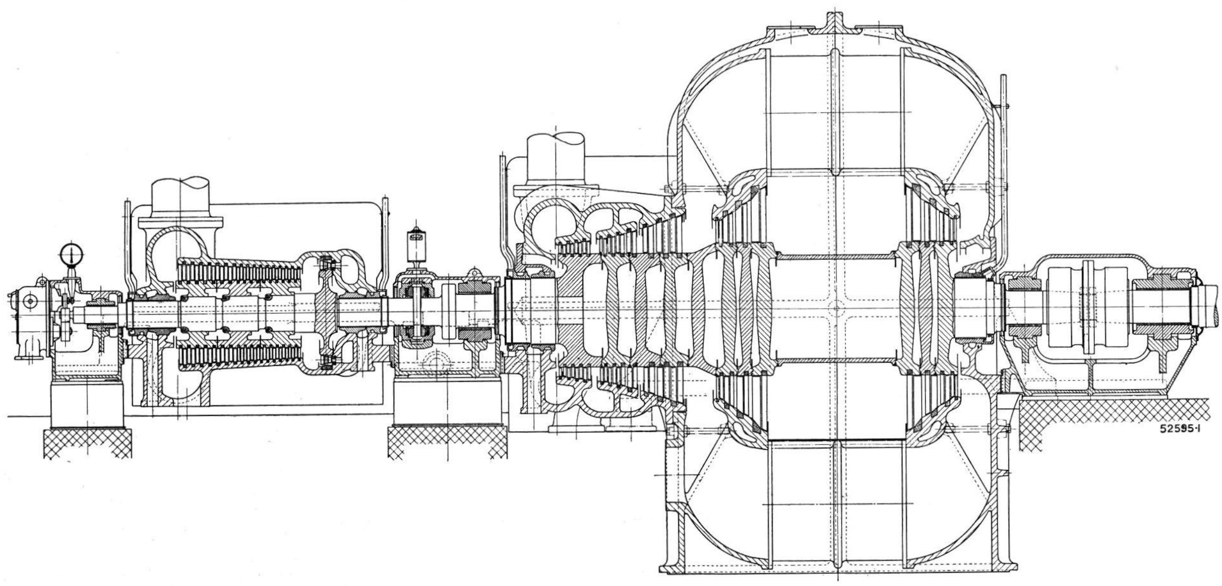


Fig. 14. — Two-cylinder turbine with double-ended exhaust, for 40/50,000 kW at 3000 r. p. m.
The high and medium pressure bladings are opposed in order to balance the thrust. Blade carriers in high-pressure part secured by lip welding. Low-pressure rotor built up by external welding of the shaft ends and reaction discs. Valve casing and main stop valve welded straight on to the high-pressure cylinder. — Built in 1939.

on to deliver load during long periods should have the highest efficiency possible and that it is better to use older machines which were still serviceable to cover load peaks, because the lower efficiencies of these machines were not such a great drawback as they only ran for short periods. Thus, according to our views, the new single-cylinder turbine had to satisfy both the demand for the highest efficiency and for a reasonable price.

Numerous new devices and considerable shop experience acquired in other fields were available to us for the development of high-grade single-cylinder turbines. Thus, for example, we were obliged to adopt new manufacturing processes such as welding in our development of the gas turbine, and the technology of high-grade welding was much furthered thereby. Welding work began to play a leading part in turbine construction, as well. The valve casings were welded on to the nozzle boxes, thus eliminating danger of flange leakages and heat stressing due to the different temperatures of the various parts. Further, in the case of high-pressure and high-temperature turbines, the nozzle boxes were welded into the high-pressure cylinder casing as well as to the casing of the nozzle valves (Fig. 12). Our welded rotors (Fig. 13)[1] are a departure of very great importance in rotor construction. It is only too well known that discs and drums, shrunk on to a shaft, work loose under load fluctuations of the turbine on account of the unavoidable differences in temperature and in this way rotor vibration is set up. By welding together at their rims the different parts of the rotor built up of discs or drums which are not bored for a shaft, a massive body is obtained similar to a single-piece rotor but with the difference that the welded rotor is far superior to a massive piece as

[1] The Brown Boveri Review, 1931, pages 43/45; 1936, pages 11/12.

regards stresses due to mechanical forces and heat and as regards the care bestowed on the manufacture of the different parts of which it is made up. The welding seam on the periphery of the different discs is subjected to low stressing so that the factor of safety is considerably higher than what is usually considered necessary.

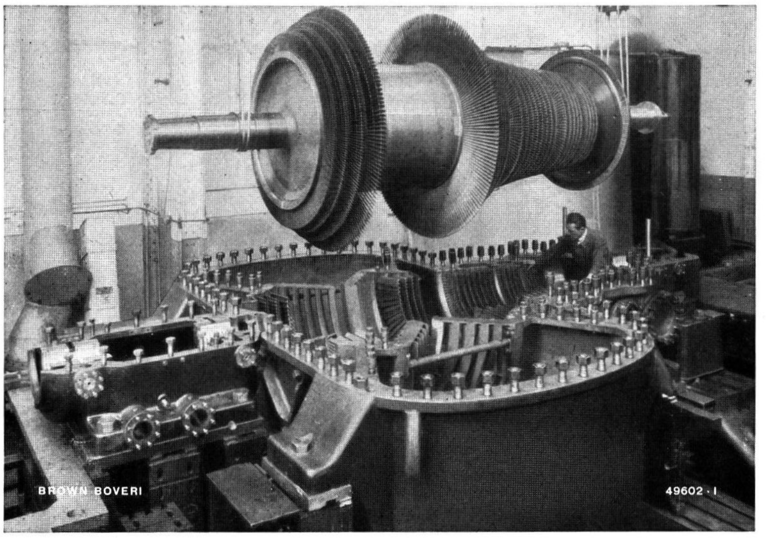


Fig. 15. — Lower part of cylinder with rotor suspended above it of a 34,000-kW, 3000 r. p. m. single-cylinder turbine with double low-pressure blading.
Built in 1939.

Formerly, the discs and balancing pistons were shrunk on and keyed to the shaft. We now use a so-termed lip weld to bind the disc to the shaft when it is necessary to use this design. The shaft and the discs have ring-shaped ridges which are welded round their periphery. This method of securing the discs is both simple and reliable and does away with the weaknesses of former designs.

As was said before, turbines with big rotor and cylinder diameters are always more expensive than those which are relatively long but of smaller diameter. For this reason, we also developed, besides our single-cylinder single-ended turbines, other single-cylinder two-ended turbines[1]. As apart from doubling the exhaust section it is thus possible to increase the section, this design is very advantageous for very big outputs and high vacuums. Of course this double-ended design can also be used for multi-cylinder turbines as well (Fig. 14)[2]. The dividing of the steam flow generally takes place at about atmospheric pressure. The steam is carried over to the second exhaust end through two pipes lodged inside the roomy exhaust housing. In order to lighten the construction of this housing, the second turbine and generator bearing are carried directly by the frame (Figs. 14 to 16).

By doubling the low-pressure part, that is with four low-pressure parts, an output of 100,000 kW at 3000 r. p. m. can be obtained,[3] the exhaust losses being kept down to an admissible figure.

It is obvious that the problem of blade fixation kept step with development of the other parts of the turbine, a development which was, at times, very rapid. As regards the way in which the blades were secured we began by adopting the Parsons design and improved on it as time went on and requirements became severer. In 1917, a very important step was taken, namely the introduction of the T root for blades (Fig. 17). This T blade is pressed out cold from the already drawn profile[4]. Contrary to nearly all other methods of securing blades, the shaft of the blade with T foot is not reduced at any point on its length, there are no notches to weaken it or sudden changes in its section.

In so far as the stressing is not to severe, we use blades of drawn cartridge brass as this material, apart from possible erosion phenomena, has given the best

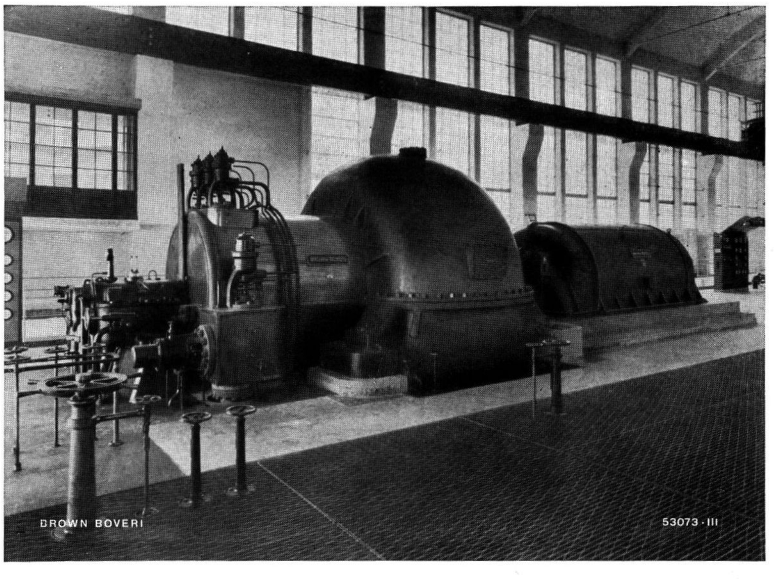


Fig. 16. — Single-cylinder turbo-set, 30,000 kW, 3000 r. p. m. with double-ended exhaust-steam blading.
Built in 1939.

[1] The Brown Boveri Review, 1936, page 10.
[2] The Brown Boveri Review, 1940, pages 224/227.
[3] The Brown Boveri Review, 1937, page 15.
[4] The Brown Boveri Review, 1924, page 179.

results. For high temperatures, we use chromium-plated steel or special steels, generally as T-root blades of drawn material. In wet steam that is in the low-pressure blading we use martensitic rustless steel. As this steel can be hardened, we increase the life of the blades by protecting them against erosion through hardening the edges on the steam inlet side.

The high temperatures and very variable heat drops often encountered gave rise to great difficulties, especially in high-pressure turbines. Exhaustive investigations with special testing machines of our own supplied us with data on the creep limit of forged and cast parts of special steel. Special methods of nozzle and blade manufacture and suitable application of welding allowed us to overcome the difficulties due to extreme steam conditions.

The problem of draining away water is closely allied to that of blading. The higher the steam pressure the lower the expanded steam penetrates into the damp steam region. Drops of water form which erode the blades by impinging on them; these drops also exercise a certain braking action. Efforts must, therefore, tend both towards protecting the blades against erosion and carrying away the water formed. Heating of feed water by extraction from the turbine allows of utilizing big quantities of condensed water along with the steam used for this purpose; the turbines are also provided with special devices to trap and carry off the drops of water formed in the turbine. We have developed a number of useful devices to this end and put them to practical application (Fig. 18)[1].

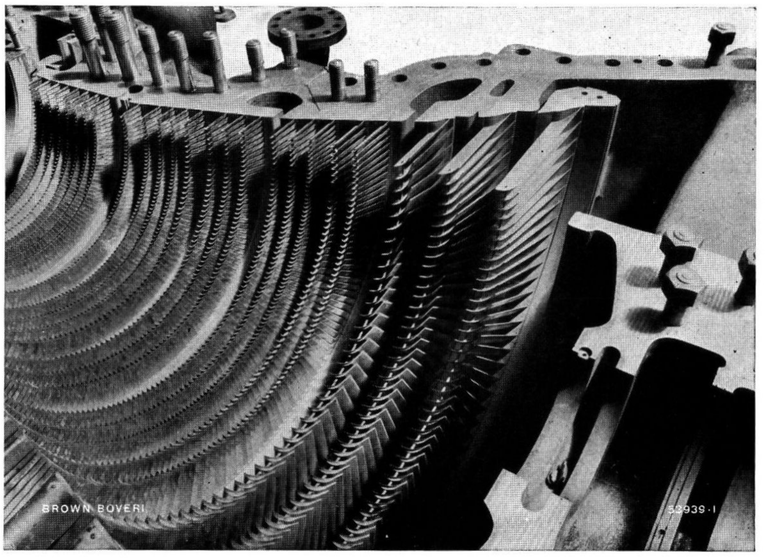
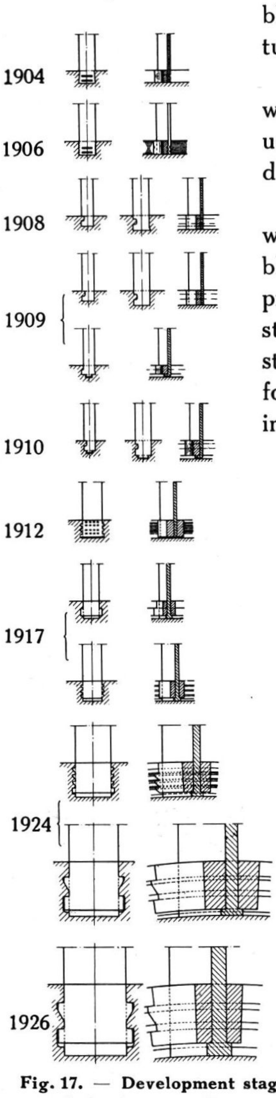


Fig. 18. — View of lower part of a cylinder with guide blades built in.
Ducts in the cylinder for steam extraction for purposes of heating the feed water and drainage. — Built in 1940.



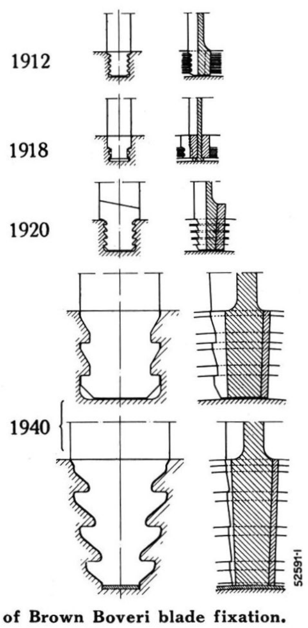


Fig. 17. — Development stages of Brown Boveri blade fixation.
Left: drawn profile. Right: milled profile.
Up till 1917, fixation of blades by calking or notching the root. After 1917 fixation by T foot. For milled blades the blade root was thickened as compared to the shaft and notches cut in it.

It is interesting to ascertain in what measure the forty years of development of the Brown Boveri turbine has influenced the amount of material used in the machine, a factor of very great influence on the price of the plant, or, expressed otherwise, how many kW of turbine output could be got out of a ton of material in the course of the years we have been building turbines. If we plot the output per ton of material in function of time, we get a practically straight rising line. This rise is due to progressively better utilization of materials. The fact that the line is practically straight shows that the different designs, whether single-cylinder or multi-cylinder, whether single-ended or double-ended, have little influence on the weight; progress is due in much greater proportion to constant improvement of individual parts of the machine and better utilization of the materials employed.

The particular conditions in Switzerland (no raw material at all and no market for steam turbines) soon forced us to seek solutions other than those open to builders in countries amply provided with the neces-

[1] The Brown Boveri Review, 1930, page 35; 1934, page 31.

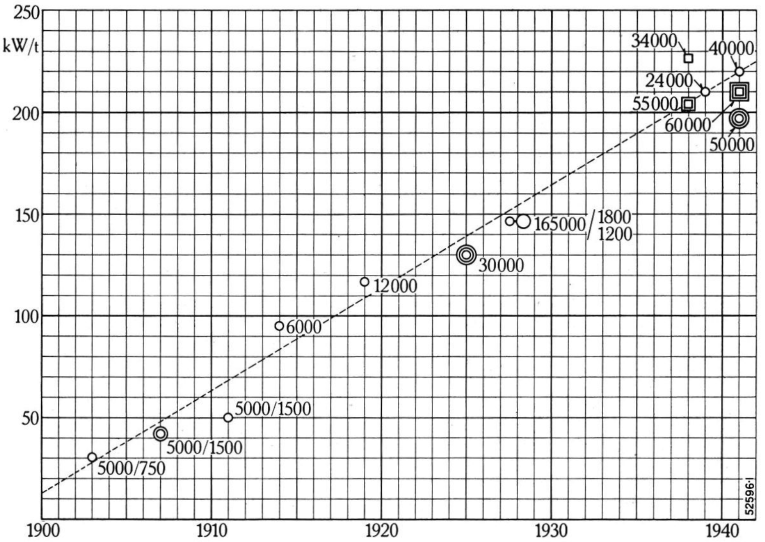


Fig. 19. — **Increase in output per ton of weight of the turbine including bedplate (without generator or condensing plant) since 1903.**

Single circle	Single-cylinder turbines.
Double circle	Two-cylinder turbines.
Triple circle	Three-cylinder turbine.
Single square	Single-cylinder turbine with double-ended l. p. blading.
Double square	Two-cylinder turbine with double-ended l. p. blading.
Triple square	Three-cylinder turbine as previous two-cylinder turbine but with second l. p. cylinder making in all 4 l. p. bladings.
Small circle—big circle adjacent	Hellgate two-shaft turbine.

sary raw materials and which had tarif-protected markets for the sale of their turbines. The necessity of being thrifty with material and of building light machines in order to reduce imports as well as to bring down freight and duty on the machines exported, has had excellent economic and technical results because the turbines being light for their output were subjected to lower heat stressing and were more flexible in following changing operating conditions. It is a satisfaction to us to be able to assert that the Brown Boveri turbine has always ranked as a model of its kind and gained the confidence of the technical world thanks to its economic operation and service reliability.

It can be said that having a Brown Boveri turbine in his power house is a source of pride to many a station operating engineer.

(MS 802) *K. Frey. (Mo.)*

ON THE FIRST STEPS TOWARDS THE UTILIZATION OF VERY HIGH STEAM PRESSURES.

Decimal index 621. 1-174. 1 (091)

To-day, very high pressure steam, superheating to high temperatures and heating of feed water by extraction steam are processes utilized wherever it is considered advantageous to do so. Their application no longer means the solution of difficult problems. Things were different, however, two decades ago. The object of the following retrospective article is to show that we did pioneering work in laying the basis of high steam pressure engineering and were, indeed, the first to put high pressures to practical use.

THE interest aroused in certain countries, shortly after the World War, in increasing the steam pressures and temperatures used in steam power stations, was chiefly due to a desire to counterbalance, as far as possible, the abnormal rise in the cost of fuel by bringing down the operating costs of the plant. It had also long been the aim of designers to increase the pressure of the steam used in steam engines. Both Perkins and Alban proposed doing so, in the twenties of the last century, and the problem was tackled again towards the beginning of the present century. The practical means of solving the problem were, however, still insufficient at that time. The first practical attempt to go considerably beyond the standard pressure range of 12 to 16 kg/cm^2 was made in England in 1917, when a pressure of 33 kg/cm^2 was specified for the North Tees power station, begun in that year. This was a marked departure from standard customs, although it did not reach, by far, what we may really term the very high pressure range, namely pressures of over 50 kg/cm^2.

Our own first serious studies into the problem of very high pressures date back to 1920. At first, these were purely mathematical conceptions, carried out with an extrapolated entropy table. Being convinced that it was less the steam turbine itself than the steam boiler which would put a limit on the practical steam pressures which could be used, we circulated all the more important boiler makers in December 1920 to find out what their standpoint was. The replies we got did not deviate much from one another. Conservative firms recommended remaining below the 20 kg/cm^2 limit on account of "considerable difficulties", the majority were for 25 kg/cm^2. The British firm Babcock and Wilcox, who had built the North Tees boilers, gave 33 kg/cm^2 with 400^0 C maximum temperature of the superheated steam as the utmost which they could recommend. The very cautious head of a well known firm amplified his written communication with a telephonic one to the effect that his firm would feel much more comfortable if 10 kg/cm^2 were not exceeded.

Thus, there was not much to be hoped from boiler makers, although it must be remembered that improvement of the plant overall efficiency did not depend, solely, on the steam pressure. Thus, the increase in the temperature of superheated steam gave promise of considerable gains. If we confined ourselves, at first, to this last problem and took out a number of patents, at the beginning of 1921, on turbines with high superheating and separately fired superheaters, this may have been due to the problems set by the development of the gas turbine to which we were then devoting considerable attention. Apart from this, however, we made studies for extraction-steam feed heating up to saturation temperature and studied preheating of air for boilers made possible — or rather necessary — by the extraction-steam feed heating.

About the middle of 1921 Hartmann published the work carried out by the Schmidt'sche Heissdampfgesellschaft with a steam boiler for 50 kg/cm^2.[1] This was a lead to boiler makers and gave a fresh impulse to our own efforts in the field of high pressures. A whole series of plans for very high-pressure steam turbines were drawn up and many calculations made in order to prove the economy of very high-pressure plants in conjunction with superheating to high temperatures and heating of the feed to high temperatures by extraction steam. The first clients before whom we were able to lay our plans and to whom we tendered a high-pressure plant of 50 kg/cm^2 and 480^0 C; were the managers of the American Gas and Light Co., who visited us in April 1922. The realization of this plant was conditional on the clients finding a suitable boiler. As far as we knew, practically nothing had been done in America, up till then, as regards high-pressure steam, apart from some discussions between experts interested in the subject.[2]

Our work got a fresh impulse in July 1922 when the Swede Blomquist came to Switzerland. He had been an earlier collaborator of Laval and, along with the latter, had devoted careful study to very high steam pressures as early as in the nineties. This work had led him to the conclusion that standard boiler designs could not be used for very high pressures, because the necessary water circulation could not be attained in the narrow water tubes as demanded by high pressure. Thus, a type of steam boiler was evolved in which the evaporator tubes were composed of rapidly revolving drums in which the natural circulating process in the water tubes of the boiler and the separation process of the steam from the water in the drums was replaced by a centrifugal action[3]. At that time there had been several of these "Atmos" boilers operating in Sweden since a considerable time and working to pressures of 50 and 60 kg/cm^2. These had given good results. Our hope of getting suitable boilers for the developments we had in mind, was strengthened when we learnt that the Sudenburger Maschinenfabrik in Magdeburg had acquired the licence rights for building the Atmos boiler and were devoting their full attention to the further improvement of the boiler in question.

At that time a scheme was on foot to enlarge the old Rummelsburg electricity works near Berlin. This plant contained old turbines which were still in good condition. It was, therefore, tempting to try to reach the bigger power output by increasing the steam pressure, which would have allowed of utilizing the existing plant. In the meantime our studies and plans for very high-pressure turbines[4] and preliminary work on the boilers were so far advanced that we could risk putting forward a very high pressure plant for the enlargement in question; the very high pressure steam was to be used in primary turbines to cover the basic load, the steam being expanded in the said turbine and then led to some of the existing low-pressure turbines, where it was further expanded, while the remainder of the low-pressure turbines in conjunction with the old boilers served to cover the peak load.

Our tenders, put forward in March 1923, are interesting because they contained already everything which was adopted only step by step later in connection with very high pressure steam. The pressure was 100 kg/cm^2, the temperature of the superheated steam 460^0 C, the output of each primary turbine 10,000 kW, the back pressure of the primary turbines, which is also the pressure of the existing plant, 15 kg/cm^2. The exhaust steam from the primary turbines was to be superheated again in flue-gas superheaters. Extraction steam heating of the feed water to 230^0 C and a high temperature of air preheating for the boilers was provided. A kind of storage medium pressure container was used as compensator between the high and the low pressure systems. There were to be 7 Atmos boilers, each for 16 t of steam per hour, with 16 rotors.

This scheme was never carried out. In June 1923, it was decided to shut down the old Rummelsburg plant entirely and build a new power station. Thus, the Klingenbergwerk power station came to be built, for which a steam pressure of 40 kg/cm^2 was considered sufficient.

We then found another occasion to put up a high-pressure plant. M. Herry, Director of the Centrales Electriques des Flandres in Langerbrugge in Belgium had become interested in our schemes as early as December 1922 and grew enthusiastic as to the possibilities they offered. He decided to give them practical expression in the extension to be made to his power station. The project prepared in collaboration with us was for a steam pressure of 84 kg/cm^2. When, however, the plant was begun in August 1923,

[1] O. A. Hartmann:—"Hochdruckdampf bis zu 60 atm. in der Kraft- und Wärmewirtschaft", Zeitschrift VDI 1921, page 663.

[2] Orrok:— "The Commercial Economy of High Pressure and High Superheat in Central Power Stations." Power 1922, pages 684/913.

[3] E. Josse:— "Hochdruckdampferzeugung durch Atmoskessel", Zeitschr. VDI 1925, page 169.

[4] W. G. Noack "Hochdruck und Hochüberhitzung", Zeitschr. VDI 1923, page 1153.

it was decided to go to 56 kg/cm^2 boiler pressure only. Agreement on this pressure was due to the fact that it was the maximum which allowed operation without intermediate superheating, assuming a temperature of superheated steam at turbine inlet of 450^0 C, which was the maximum Messrs. Babcock and Wilcox the boiler makers would guarantee. In the Langerbrugge plant the heating of feed water by steam bled from the turbine was carried further than in any preceding plant. It reached 200^0 C. The combustion air for the boiler is preheated to the same temperature.[1]

The plant was put into operation for the first time in November 1925. It was the first very high pressure plant in the world to be put up in a public utility electrical power station. The progress which it incorporated was the more cheering because it was attained without a single setback. This was, indeed, a pioneering effort, it showed clearly how existing power stations could be operated at lower cost with the help of very high steam pressures, primary turbines and the new operating processes. As a matter of fact, the overall efficiency of the Langerbrugge plant was improved from about 17 % to about 23 %.

During the year 1923, our studies on high-pressure steam were published for the first time. A great sensation was caused when we gave a verbal report on our work, at Weimar on the 28th and 29th June 1923 on the occasion of the Meeting of German electricity works engineers, on which occasion, we handed to all members present our report on very high-pressure steam which had already appeared in the BBC Review Mannheim.[2] Still more important than this propaganda work, which was really hardly necessary for the cause of high pressure alone, was the explanatory work we carried out for the benefit of manufacturers of boilers, fittings and air preheaters. The building of suitable boilers in itself made severe demands on the boiler makers but the special operating conditions which were necessary in order to allow of taking full advantage of high steam pressures set the boiler firms problems to solve which had never been encountered before. For example, the feed water heaters heated by exhaust steam were already known, but it was a new departure to heat feed water up to 150 and 200^0 C outside the boiler by means of steam bled at several points from the turbine.[3] It took boiler makers a long time to get reconciled to this idea. Along with the heating of feed water to high temperatures, it became absolutely necessary to have a preheater for the boiler combustion air in order to utilize the heat contained in the flue gases. It was part of our pioneering work to make the firing of boilers and the grates suitable for the higher combustion-air temperature and to convince boiler makers of the necessity of these modifications.

When, however, the first results of practical operation were available, it was found that everything went easier than had seemed likely before and when, also, the first reports from America on successful plants operating with very high pressures came in, it was possible to consider a stage in the development and introduction of very high steam pressures as having been accomplished. The first pioneering work had been done.

(MS 803) *Dr. W. G. Noack. (Mo.)*

[1] W. G. Noack:— "Dampfturbinen für hohen Druck", Zeitschr. VDI 1926, page 711.

[2] W. G. Noack:— "Die BBC-Hochdruckturbine für Dampf von 100 atm. und 450^0 C Überhitzung". BBC Mitteilungen Mannheim, Heft 5, May 1923.

[3] W. G. Noack:— "Verfahren der Anzapfdampfvorwärmung", Zeitschr. VDI 1925, page 1004.

BRIEF BUT INTERESTING

ON the occasion of the recent overhaul of a 16/20,000-kW Brown Boveri three-cylinder turbine in Belgium, it was found that all the work required was to clean the lubricating and governing oil system and to replace some minor details. This turbine was first put to work in 1931 and had run 72,000 hours practically without being overhauled.

*

THE measurements carried out during the taking-over tests on a Brown Boveri two-cylinder turbine with double-ended low-pressure part, of 30,000 kW, 45 kg/cm^2 abs, 435^0 C, delivered to a Dutch plant, showed a thermal efficiency referred to the useful output measured at the generator terminals of 31·6 %, this being an average value for the output range of 10 to 100 % load. 31·6 % efficiency means 2754 kcal heat consumption per kWh produced. This exceptionally low heat consumption figure measured by an acceptance test commission, in collaboration with the client, is 1·86 % below the figure guaranteed with the order.

NUMEROUS investigations (carried out chiefly by Witte) showed that the efficiencies of modern high-pressure back-pressure turbines of a great variety of designs differed but little from one another. Lately, measurements were made on an 11,000-kW Brown Boveri back-pressure turbine which was put to work in May 1940; its efficiency proved to be very considerably higher than what had been obtained before. Referred to output on the coupling, it was 82 %. Closer details can be obtained from a report by Liceni entitled "Vorschaltturbine mit hohem Wirkungsgrad" in the "Archiv für Wärmewirtschaft", No. 7 of 1941.

*

WE have often mentioned our high-pressure turbine delivered to the Caroline Pit (Witkowitz) which we like to point out as proof of the exceptional reliability of Brown Boveri turbines. This high-pressure condensing turbine of 36,000 kW output, 120 kg/cm^2 abs, 500^0 C has been in service since 1933 and has run for 8000 hours per year, on an average, since it first took over load.

THE END LOSSES OF TURBINE BLADES.

Decimal index 621.165.018
621.165:533.6.01

Additional losses occur at the ends of turbine blades, which exercise a considerable influence on the efficiency of turbine blading.

These end losses are only partly dependent on the clearance. This is shown:

1. With the aid of a formula due to Betz for the induced drag of a wing with fluid flow in a clearance gap at the free end of the wing.

2. By examination of the influence of the secondary flow at the ends of blades without clearance.

3. By tests on grid models in which the secondary flow is made visible, and

4. By tests on an air-turbine from which the magnitude of the end losses is calculated for a single-stage reaction turbine.

A new formula is suggested for the efficiency of turbine blading which takes into account the theory of similarity.

The constants of the new formula were calculated from the measured efficiencies by the method of minimum squares of Gauss. The formula agrees well with the measured values.

I. INTRODUCTION.

At some distance from the ends of the blades of a grid, only the pure profile losses occur, but at the blade ends the losses increase to such an extent that the efficiency of a turbine is to a high degree determined by these end losses.

The end losses are in the first place dependent on the shape of the blade end, i. e. on the profile together with any sharpening of the blade tip, on the pitch and on the clearance. In the second place, the end losses are fundamentally dependent on the type of flow and consequently on the Parsons coefficient, on Reynolds' and Mach's numbers as well as on the degree of turbulence before the blading.

Of all these influences, actually only that of the *clearance* has been more closely observed up to the present time. This is easy to understand when it is considered that the influence of an increased clearance on the efficiency makes itself felt very clearly and in an extremely unpleasant manner, so that engineers concerned with the practical construction of turbines had to study this question from the very beginning.

On the other hand, it is not so easy to see that this *clearance loss* by no means represents the *total end loss*. For this it is first necessary to know the actual profile efficiency of the blading, and it is not at all simple to measure the actual, pure profile efficiency on a completed turbine. Consequently the clearance was regarded as the only source of end losses and the efficiency, obtained by extrapolation to zero clearance, was taken as the profile efficiency.

In the literature dealing with this subject it is practically never mentioned that considerable losses also occur at blade ends without clearance, e. g. on shrouded blading, on nozzle blades and on bladed rings as found in the Ljungström construction. That even Prof. Stodola in his well-known book does not touch on this question at all, may serve to show that such thoughts did not even enter the heads of the leading men in the field of steam turbine construction, because Stodola's work "Dampf- und Gasturbinen" contains, even to-day, a veritable fund of interesting and valuable observations and suggestions. The English "Nozzle Committee", which carried out tests on nozzle segments over a number of years and which had adequate means at its disposal, also preserves an absolute silence over this question in its reports, a fact which is all the more surprising when it is considered that the "Nozzle Committee" was chiefly concerned with nozzle segments of small height, that is with blade grids with very short blades for which the end losses represent a quite considerable portion of the total losses.[1]

The modern theory of circulation has perhaps also contributed its part to veiling the end losses. In the case of an aeroplane wing, the "lifting vortex" round the wing must be cast off at the wing tip, and the trailing, marginal vortices which result, provide a very clear explanation of the tip losses. In the case of bladed grids with blades bounded at both ends, on the other hand, the circulation round individual blades, in accordance with the usual, simplified method of treatment, can close "by penetration through the end walls" external to the space where flow takes place, so that apparently no vortices at all can separate and cause end losses. The pure surface friction of the end walls is, moreover, not appreciable. This gives rise to the impression that, apart from the clearance losses, the end losses are negligible.

This opinion could be maintained so long as the actual, pure profile efficiency of the usual turbine blades was not exactly known. It has, however, already been found possible, a number of years ago, to measure the profile loss of an aerofoil, in a comparatively simple manner, by measurement of the pressure head in the wake behind the wing.[2] We have made similar measurements on models of turbine blades with considerable success.[3] In the course of these tests it soon became clear that the profile efficiencies determined in this manner were considerably higher than those obtained from turbine tests by extrapolation to zero clearance. The difference could only be due to additional end losses, and it was consequently necessary to examine this question more thoroughly.

[1] 50 % and more.
[2] See, for example, Durand "Aerodynamic Theory".
[3] Brown Boveri Review, January/February 1938.

II. THEORY.

A great deal has been published on the tip losses of aeroplane wings with free ends, but the sources of information for aerofoils with a solid boundary at their ends are few and far between. Nevertheless, an investigation carried out by Betz[1] yields really interesting information.

Betz gives the following formula for the *minimum* induced drag due to flow in a clearance gap at the tip of a wing

$$W = k_2 \cdot \frac{A^2}{\varrho \cdot w^2/2 \cdot l^2} \tag{1}$$

W = induced drag
A = lift
l = blade length
w = mean, relative velocity of flow
ϱ = density

k_2 is a form factor which can be calculated from the following formulae:—

$$k_2 = \frac{K(x)}{8 \cdot K'(x)} \tag{2}$$

where $K(x)$ and $K'(x)$ are the complete, elliptic integrals

$$K(x) = \int_0^{\frac{\pi}{2}} \frac{d\varphi}{\sqrt{1 - x^2 \sin^2 \varphi}} \tag{3}$$

$$K'(x) = \int_0^{\frac{\pi}{2}} \frac{d\varphi}{\sqrt{1 - (1 - x^2) \sin^2 \varphi}}$$

and where the variable x has the form

$$x = \tanh\left(\frac{\pi \cdot s}{t \cdot \sin \alpha_2}\right)$$

s = clearance
t = pitch
α_2 = relative exit angle

This formula of Betz can be converted in the following manner. Instead of k_2, a function σ is introduced which is defined by

$$\sigma \equiv \frac{\pi}{\log_e 2}\left(4 k_2 - \frac{s}{t \cdot \sin \alpha_2}\right) \tag{4}$$

The function σ disappears for $s = 0$, and increases very rapidly with increasing s, approaching the value 1 asymptotically (Fig. 1). For all practical purposes we may here put $\sigma \cong 1$, under which circumstances

$$k_2 \cong \frac{1}{4}\left(\frac{s}{t \cdot \sin \alpha_2} + \frac{\log_e 2}{\pi}\right) \tag{5}$$

[1] "Hydraulische Probleme" p. 161 seq.

If, now, formula (1) is applied to symmetrical turbine blading, we obtain by calculation the expression

$$\Delta \cong \frac{\Delta w_t \cdot \overline{w}}{u \cdot w_a}\left[\frac{s}{l \cdot \sin \alpha_2} + \frac{\log_e 2}{\pi} \cdot \frac{t}{l}\right] \tag{6}$$

where

Δ = power loss referred to the output
Δw_t = alteration of the circumferential component of the velocity
$\overline{w}$ = mean velocity $= \left(\vec{w_1} + \vec{w_2}\right)/2$
w_a = axial component of the velocity
u = circumferential velocity.

The first term in the brackets is obviously the clearance loss. The second term is an additional loss which, within practical limits, is independent of the clearance but which exists, at least theoretically, only because a clearance is actually present. This fixed end loss is proportional to the pitch.

Formula (6) can be rewritten in different forms. Two such conversions may be briefly described here, since they provide interesting fundamental conceptions.

1. Formula (6) may be multiplied and divided by l/D (D = mean diameter of the blading). Since $\pi D/t = n$ = number of blades, we have in the first place

$$\Delta \cong \frac{\Delta w_t}{u} \cdot \frac{\overline{w}}{w_a} \cdot \frac{D}{l}\left(\frac{s}{D}\frac{1}{\sin \alpha_2} + \frac{\log_e 2}{n}\right) \tag{7}$$

Further $w_a \cdot l/D = V/\pi D^2 = c/4$, where V is the volume passed in unit time, and consequently c is that (fictitious) velocity with which the volume V would flow through an opening of circular cross-section with diameter D. Since, moreover, $\Delta w_t/u = gH/u^2$ and

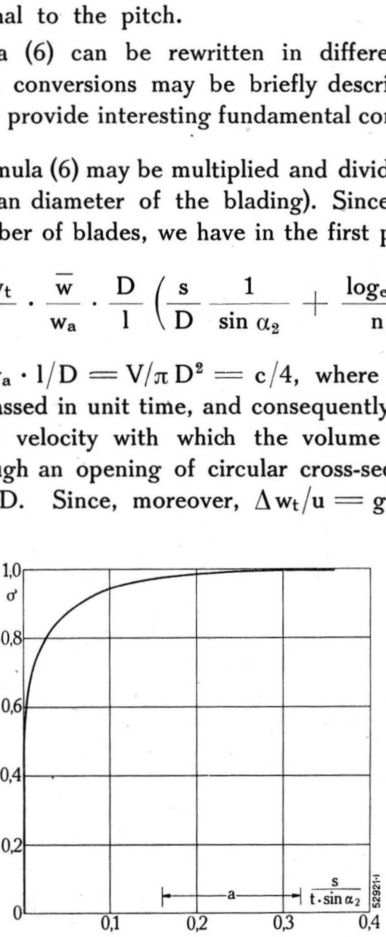


Fig. 1. — Behaviour of the function :

$$\sigma = \frac{\pi}{\log_e 2} \cdot \left[4 k_2 - \frac{s}{t \cdot \sin \alpha_2}\right]$$

The figure shows it is sufficiently accurate to put $\sigma = 1$ in the range occurring in practice (which is indicated by ←a→). It then follows that:

$$k_2 \cong \frac{1}{4}\left[\frac{s}{t \cdot \sin \alpha_2} + \frac{\log_e 2}{\pi}\right]$$

$u/2\bar{w} = \cos\bar{\alpha}$ ($\bar{\alpha}$ = mean change of direction of flow), we obtain the formula

$$\Delta \cong 2\,\frac{gH}{uc}\cdot\frac{1}{\cos\bar{\alpha}}\left(\frac{s}{D}\cdot\frac{1}{\sin\alpha_2}+\frac{\log_e 2}{n}\right) \quad (8)$$

The blade length does not appear at all in this formula. Small end losses can, therefore, only be obtained by means of:—

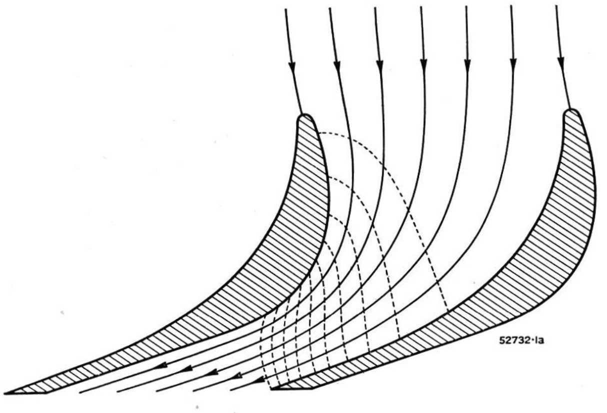


Fig. 2. — Flow in the channel between two blades.
——— = Streamlines.
········ = Isobars (lines of equal pressure).
The pressure gradient is perpendicular to the isobars. The component parallel to the streamline accelerates the flow, while the component perpendicular to the streamline diverts the flow.

(a) small heat drop per stage, H
(b) large circumferential velocity, u
(c) large fictitious velocity, c.

So far as the blade angle is concerned, the following statements apply. In the case of blades having small curvature — the calculation according to the Betz formula is only valid for such blades — $\bar{\alpha} \cong \alpha_2$. The first term in the brackets is, therefore, a minimum for $\bar{\alpha} \cong 45^0$. Since the profile losses are also a minimum for $\bar{\alpha} \cong 45^0$, the most favourable value will be somewhat less than 45^0 due to the influence of the second term in the brackets.

2. Another very interesting form is obtained if the *stressing of the blades by the centrifugal force* is introduced.

The tensile stress produced in a cylindrical blade is

$$\sigma_z = 2\,\frac{\gamma}{g}\cdot u^2\cdot l/D$$

(γ = specific weight of the blade material)

$$\frac{l}{D} = \frac{1}{2}\cdot\frac{\sigma_z}{\gamma}\cdot\frac{g}{u^2} \quad (9)$$

σ_z/γ represents here a length, and is indeed that length which, when freely suspended, would produce the same tensile stress at its upper end due to its own weight as is produced in the blade by the centrifugal force. This length — which is related to the "rupture length" — will here be represented by L. Since $w_a/\bar{w} = \sin\bar{\alpha}$, we have

$$\Delta = 2\,\frac{H}{L}\cdot\frac{1}{\sin\bar{\alpha}}\left(\frac{s}{D}\cdot\frac{1}{\sin\alpha_2}+\frac{\log_e 2}{n}\right) \quad (10)$$

Since $\bar{\alpha}$ cannot be much greater than 45^0 on account of the profile efficiency, and s/D is also fairly constant, formula (10) shows that the end losses can only be kept small by adopting a *small heat drop* and a *high stress in the blades.*

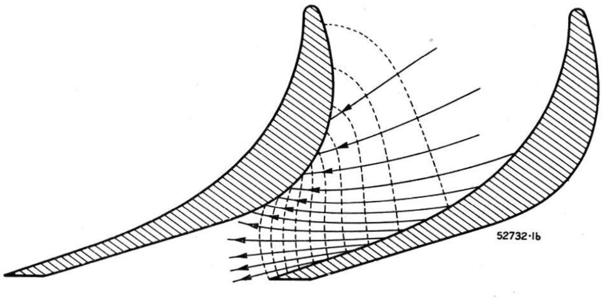


Fig. 3. — Flow at the blade end (secondary flow).
——— = Streamlines.
········ = Isobars (lines of equal pressure).
The isobars are the same as in Fig. 2. The streamlines follow the pressure gradient and are, therefore, everywhere perpendicular to the isobars.

The width of the blade appears in neither of the two formulae. Its only influence is that the number of blades n can be made larger for narrow blades without spoiling the profile efficiency.

If the clearance actually disappears completely, $\sigma = 0$ in equation (4) and the end losses likewise disappear in theory. In spite of this, tests show that considerable end losses, accompanied by the formation of vortices, still remain in existence. These end losses are due to secondary flow at the ends of the channel between the blades.

Fig. 2 shows the undisturbed streamlines between two blades, and also, as dotted lines, the lines of equal pressure. The pressure drop is at right angles to the lines of equal pressure, the component in the direction of flow producing an acceleration, and the perpendicular component causing the flow to change direction.

At the ends of the blades the velocity is reduced to zero by the friction and there is, therefore, no centrifugal force. A flow is, therefore, produced in the immediate vicinity of the wall which is chiefly determined by the pressure drop alone, and the streamlines are thus perpendicular to the lines of equal pressure in this zone. Fig. 3 shows these streamlines.

Figs. 4 and 5 show that this secondary flow has a similar action to a clearance gap at the end of the blades. Fig. 4 represents the free end of a blade. Since the pressure on the hollow side of the blade is greater, flow takes place round the free end of the blade. The streamlines on the hollow side spread out like a fan, while on the back of the blade they are forced away from the end. The streamlines com-

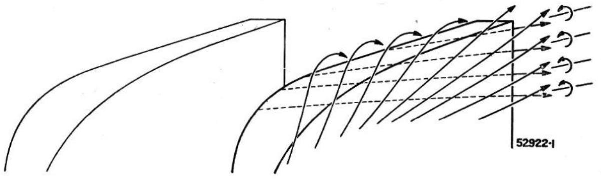


Fig. 4. — Flow at the free end of a blade.
Flow takes place round the end of the blade because the pressure on the hollow side is greater than the pressure on the back of the blade. The streamlines spread out like a fan on the hollow side, and are forced together on the back. At the trailing edge the two flows cross one another and thus form trailing vortices.

ing from both sides of the blade are no longer parallel at the trailing edge, but cross one another. Vortices are, therefore, formed and cast off at the trailing edge, thus producing end losses.

Fig. 5 shows the influence of the secondary flow at the end wall of the channel between two blades. For the sake of clearness, the end wall itself has been left out of the diagram. As may be seen, the diagram of the streamlines on the blade surface is quite similar; on the hollow side the streamlines spread out like a fan while on the back of the blade they are forced away from the blade end. The streamlines, however, are no longer closed over the clearance gap, but by way of the secondary flow along the end wall. The effect is practically the same; at the trailing edge vortices are formed which cause end losses.

In both cases further losses occur due to the introduction of an alien flow on the back of the blade, which can, under circumstances, cause the flow to separate from the back of the blade. These losses cannot, however, be treated theoretically.

III. TESTS.

The flow at the ends of blades can be easily observed on stationary models. In addition to the usual observations with Pitot tubes and streamers, the following

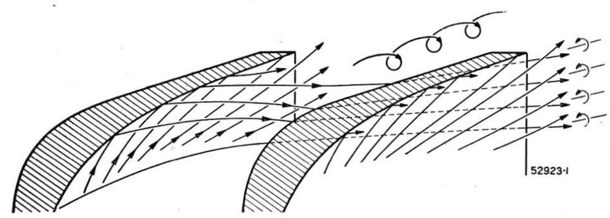


Fig. 5. — Secondary flow at the end of a channel between two blades.
The end wall has been removed in order that the flow may be more clearly illustrated. The streamlines on the hollow side are drawn out to the form of a fan by the secondary flow, and on the back they are forced together. At the trailing edge the streamlines cross one another and form trailing vortices exactly as in the case of the free blade end.

method has proved to be very instructive. The blade is given a coat of a suitable oil paint, and is then subjected to the action of the air current while the paint is still wet. The paint is partly blown away and the streamlines in the immediate neighbourhood

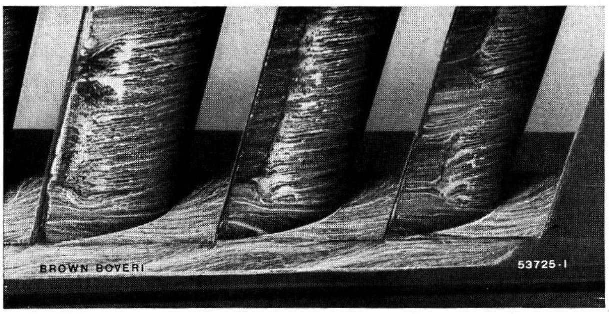
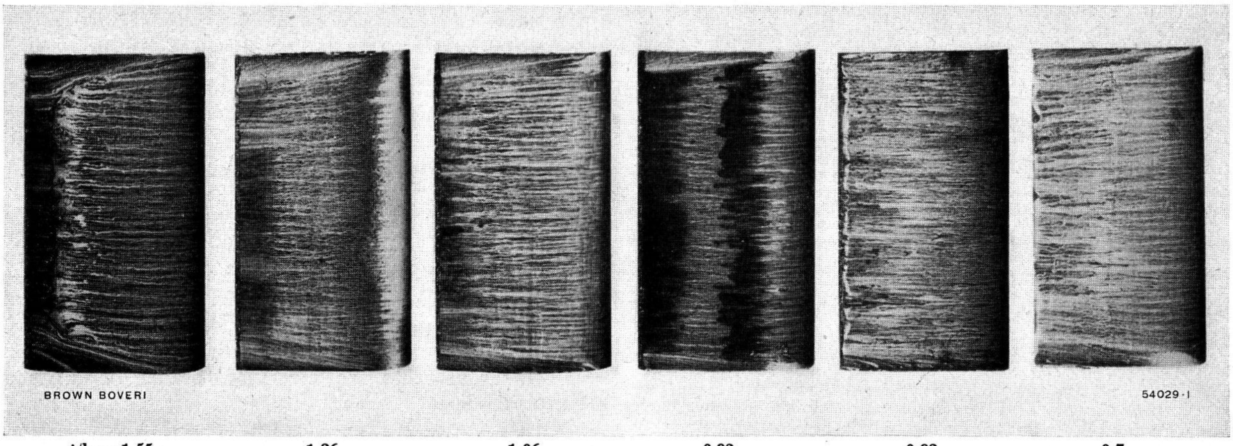


Fig. 6. — Secondary flow at the end wall of a grid of blades.
The streamlines in the immediate vicinity of the wall are recorded on the coat of wet paint. On the back of the blades, the streamlines are forced together. It is noteworthy how exactly the streamlines at the end walls conform with the theoretically deduced streamlines shown in Fig. 3.



$t/b = 1{,}55$ 1,26 1,06 0,92 0,82 0,7

Fig. 7. — Increase of the end losses with increasing pitch.
While the end interference is hardly visible for the smallest pitch, it extends far into the healthy flow in the case of the largest pitch.

of the walls are clearly reproduced in the wet paint on the blades and intermediate pieces. In this manner, the secondary flow at the blade ends, as deduced from the theory given above, can be rendered visible during the test (Fig. 6).

In the accompanying photograph (Fig. 7), the disturbed end zones can be distinctly seen in the coat of paint. When the pitch is small, only a very narrow interference zone is formed, which increases rapidly with increasing pitch. Although no conclusions can be drawn from these pictures as to the magnitude of the end losses, it is nevertheless quite obvious that the end losses must increase with increasing pitch, just as required by the theory.

It is not possible to determine the magnitude of the end losses with stationary models. The "induced losses" are by no means definitely lost immediately behind the wing; fundamentally speaking, they could be recovered[1]. The measurement of the energy immediately behind the blade model, e. g. with a Pitot tube, consequently yields only a very small end loss. The energy, however, is partially transformed into such a mixed state by the induced vortex, that it can no longer be usefully recovered in a normal row of blades; it is devaluated, although the degree of devaluation cannot be measured on a stationary model. Further tests were, therefore, carried out on an air-turbine and the results were evaluated on the following basis:—

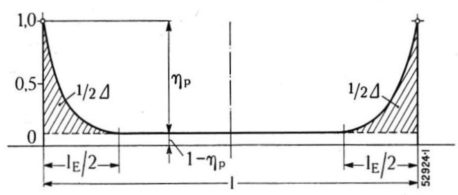


Fig. 8. — Schematic representation of the end losses.
The end losses Δ only occur at the ends over a length l_E. External to this zone, only the pure profile losses $1-\eta_P$ are to be found. If the blade is so long that the two end zones do not touch one another, i. e. if $l > l_E$, the end loss is independent of the blade length. Quantity and output are linear functions of the blade length.

So long as the blade is longer than the total interference length l_E at the ends (Fig. 8), it can be assumed that the blade length has no influence on the end interference. If, therefore, two blade ends are geometrically similar as regards profile, pitch, sharpening and clearance, the interference at the end is a function of the geometrical dimensions enumerated,

[1] When migratory birds, e. g. wild geese and cranes, fly in V-formation, the trailing birds make partial use of the vortex energy left behind in the air from the flapping of the wings of the leading birds. The foremost bird does not enjoy this advantage and is, therefore, relieved from time to time.

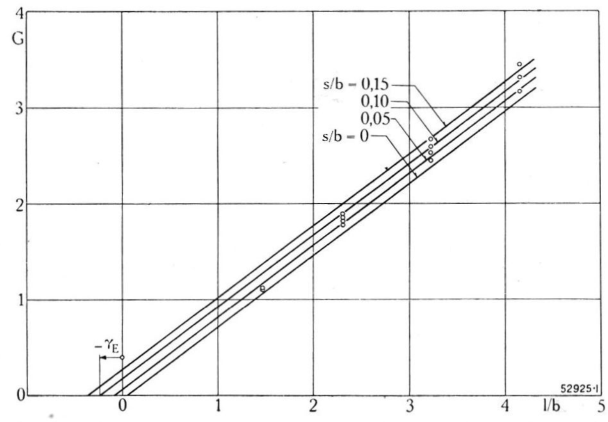


Fig. 9. — Determination of the quantity increase at the blade end.
o = measured value (see table).
The lines represent the corrected values with the blade clearance s/b as parameter.

as well as of the type of flow, i. e. it is also a function of the Parsons coefficient and of the Reynolds and Mach numbers, but it is *independent of the blade length.* For otherwise identical conditions, both the quantity G passing through the turbine, and the output L are linear functions of the blade length, and the following expressions are valid:

$$G = g_b \cdot l/b + G_E \qquad (11)$$

$$L = G \cdot H \cdot \eta_P - L_E \qquad (12)$$

l = blade length
b = blade width (serves as unit of length)
G = quantity flowing past the blade
G_E = additional quantity at the blade end
g_b = quantity per unit of length
L = output
L_E = end loss
H = heat drop
η_P = profile efficiency

With the abbreviations

$$\gamma_E = G_E/g_b \qquad (13)$$

$$\delta_E = L_E/g_b H \qquad (14)$$

we obtain the following expression for the efficiency:—

$$\eta = \eta_P - \frac{\delta_E}{l/b + \gamma_E} \qquad (15)$$

The quantities δ_E (end loss) and γ_E (end quantity) are functions of the shape of the blade end and of the type of flow, but are *independent of the blade length.*

If measurements are made on blades of different lengths, which are otherwise the same, δ_E aud γ_E can be determined. The measured values of G are first plotted in function of l/b, in order to find γ_E (Fig. 9). Then the measured values of the output are likewise

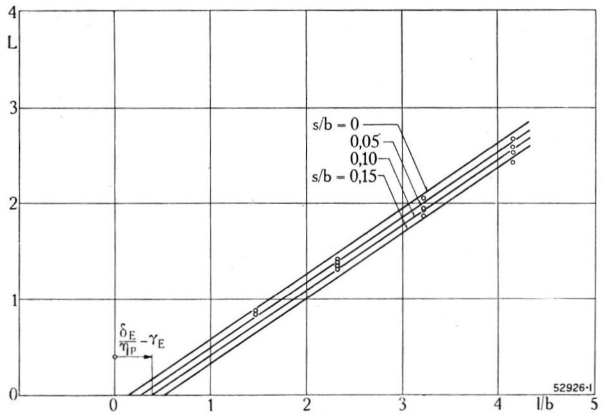


Fig. 10. — Determination of the reduction of output at the blade end.
o = measured value (see table).
The lines represent the corrected values with the blade clearance s/b as parameter.

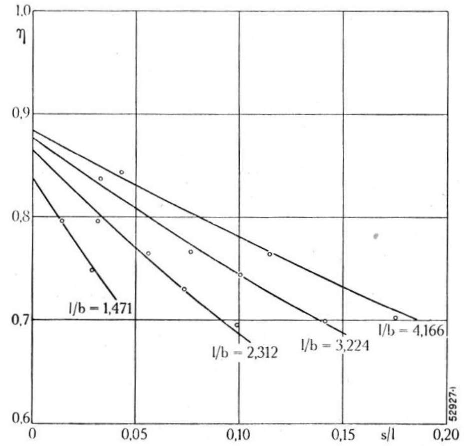


Fig. 11. — Comparison of the efficiencies as measured and as calculated from the formula.

$$\eta = \eta_p - \frac{\delta_E}{l/b + \gamma_E}$$

$$\eta_p = 0{\cdot}9091 \pm 0{\cdot}0069$$

$$\gamma_E = -0{\cdot}0422 + 2{\cdot}790 \cdot s/b$$
$$\pm 0{\cdot}0037 \pm 0{\cdot}074$$

$$\delta_E = +0{\cdot}1011 + 4{\cdot}667 \cdot s/b$$
$$\pm 0{\cdot}0118 \pm 0{\cdot}158$$

The simplest possible equations were chosen both for γ_E and δ_E. The agreement with the measurements is good.
(The numerical values are only valid for the single-stage reaction turbine tested !)

plotted in function of l/b in order to obtain δ_E and η_P (Fig. 10).

On account of the unavoidable errors in the observed values, it is definitely well worth while to employ the method of least squares for correcting the readings, because this method gives not only the most probable values of the desired quantities but also the mean value of the error. The correction by the Gauss method thus provides protection against an unjustified faith in the exactness of the results obtained. The table contains a complete series of measurements on a *single stage* of reaction blading. The efficiency has been determined in function of the Parsons coefficient for four different blade lengths.

Quantity [1] G	Output [1] L	Blade length [2] l/b	Clearance [2] s/b	Efficiency η
3·170	2·672	4·166	0·0430	0·843
— [3])	2·567	4·166	0·0801	— [3]
3·315	2·529	4·166	0·1148	0·763
3·455	2·425	4·166	0·1758	0·702
2·445	2·047	3·224	0·0330	0·837
2·535	1·942	3·224	0·0770	0·766
2·595	1·931	3·224	0·1008	0·744
2·670	1·866	3·224	0·1414	0·699
1·775	1·413	2·312	0·0316	0·796
1·815	1·387	2·312	0·0563	0·764
1·850	1·351	2·312	0·0738	0·730
1·890	1·314	2·312	0·0992	0·695
1·105	0·880	1·471	0·0143	0·796
1·112	0·838	1·471	0·0291	0·748

[1] Values, for purposes of comparison, reduced to unit heat drop.
[2] b is the blade width and serves as the unit of length.
[3] The measurement of quantity proved to be inexact.

Fig. 11 shows the results obtained by calculation from the observed data. Despite the fact that the simplest possible equation was assumed, the measured points lie in very close proximity to the calculated curves.

Especially noteworthy is the expression found for the end losses

$$\delta_E = 0{\cdot}1011 + 4{\cdot}667 \ s/b.$$

Since s/b, in practice, varies between the limits 0·04 and 0·08, the pure clearance loss, i. e. the loss dependent on the clearance, actually represents only about three quarters of the total loss. There remains, therefore, a quite considerable *pure end loss* independent of the clearance, which appreciably reduces the efficiency of short blades even for the smallest clearance.

As previously mentioned, the end losses are generally first irretrievably lost in the next row of blades. Consequently it is not possible to apply the results of tests on single-stage turbines directly to multistage turbines, but tests must be carried out for different numbers of stages, from the results of which the losses of multistage turbines may then be obtained by extrapolation.

(MS 804) *A. Meldahl. (D. S.)*

MULTIPLE GOVERNING SYSTEMS FOR STEAM TURBINES.

Decimal index 621.165.63

Our multiple-governing systems for steam turbines have been subjected to considerable transformation in the course of recent years. Under the term multiple governing system we mean one which is made up of several systems which mutually influence one another according to the duty they have to perform, as, for example, the governing of extraction-steam turbines or two-pressure turbines.

GOVERNING CONDITIONS.

AMONG all the turbines which interest us, here, and of which the multi-extraction multi-pressure turbine according to Fig. 1 is the one encountered most frequently, the only fact of interest from the purely governing point of view is that there are a number

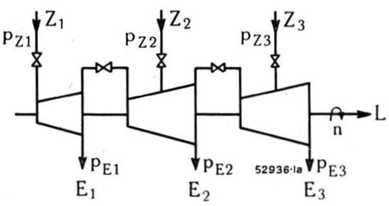


Fig. 1. — Diagram of a multiple extraction multiple-pressure turbine.
E, Z and L are operating magnitudes and p_E, p_Z and n are the governed magnitudes related thereto.

of *operating magnitudes* such as the output or the amount of steam extracted, to each of which a *regulated magnitude* is related such, for example, as the speed or the extraction pressure. While the operating magnitudes are always fluctuating, the regulated magnitudes must be maintained at constant and desired values as well as possible. It is the duty of the governing system to see that this is accomplished. The degree of perfection with which this is done is a measure of the quality of the governing system employed. In order to be able to establish a scale with respect to the quality of a governing system we have first to establish what can be fundamentally expected of a multiple governing system. The result of this investigation can be summarized in the three following requirements:—

1. It is a fundamental characteristic of every kind of governing system that the exact value desired can never be exactly maintained. The regulated magnitudes always change slightly with the operating magnitudes, if, however, within much narrower limits. If, now, a modification of the regulated magnitude along with the operating magnitude to which it is related is not only unavoidable but often even desirable, every effort should be made, on the other hand, to prevent the other operating magnitudes being affected, because it is nearly always disturbing when, to take an example, the speed of an extraction turbine varies with the amount of steam extracted (a magnitude to which it is not related). *Thus it must be required of a good multiple governing system that magnitudes which are not related do not influence one another mutually.*
2. A governing system can only fulfil the duty of maintaining constant a certain number of regulated magnitudes, if it is provided with an equal number of regulating organs such, for example, as steam inlet or extraction-pressure regulating valves or groups of such valves, which are able to fulfil at any moment an order impulse emanating from the governor, i. e. can open or close and are, therefore, able to regulate. If they cannot perform these duties or only do so in one sense, then all the regulated magnitudes cannot be maintained constant. This is what occurs to a governing system when a group of valves is full open or tight closed and can, therefore, only move in counter-direction, and these operating conditions must be taken into account. Obviously, service must be kept up in spite of this and it becomes necessary to sacrifice the constant maintenance of one or other of the regulated magnitudes. *Which of these magnitudes is sacrificed is not a matter to be left to chance. It must be possible to designate it beforehand in each case according to the conditions in each particular case.*
3. The occurrence of extreme conditions of this kind in which one or other of the valve sets reaches the limit of its regulating range, is always accompanied by some irregularity in the governing process. It is clear that this irregularity must be limited, if possible, to that regulated magnitude which is now no longer susceptible to further regulation, while the other ones should be as unaffected thereby as it is possible to make them. *Above all they have to maintain their dependence on the operating magnitude to which they are related, i.e. their inherent regulating characteristic must be maintained.*

Our *new multiple governing system* fulfils these three requirements to an exceptional degree. Like our former governing system, this one is also entirely without rods. Further, the elementary parts of which it is made up, the governor and the governing valves, are of the same type as before. This also applies to the reliable system of transmitting impulses from the governor to the valves; what is new, is the introduction of a device which we term *"the pressure transformer"* and which is really the center of the whole governing system. Fig. 2 shows the fundamental arrangement. The pressures p′ regulated by the governor 1 are led to the pressure transformer 2, which in its turn regulates the pressures p″ and also the regulating valves 3, in function of pressure p′. Pressures p′ and p″ can, therefore, be designated as primary and secondary pressures. It is easy to understand that with a governing arrangement like this, it depends solely on the *pressure transformer* whether the above mentioned governing conditions be fulfilled or not; both governors and valves are simply its auxiliaries.

In principle, the *pressure transformer* is extremely simple. It comprises a casing in which a number of slide valves are lodged (Fig. 3), each of these governs the pressure in a secondary oil circuit through the agency of its regulating edges x and y. These slide valves are designed in the form of pistons of the differential type and the primary pressures act on their surfaces. The recall of the piston is produced by the regulated pressures, which are led to their upper face surface. The secondary pressures must, therefore, balance the primary pressures on the differential piston and are a definite function thereof. By suitable choice of the direction of the primary pressures and determination of the piston sections of the differential piston on which these forces act, it is possible to attain the first of the conditions we laid down above. The dimensions of the differential pistons can be deduced relatively easily from magnitudes taken from the steam-consumption diagram and can, therefore, be calculated in advance for every practical case under consideration. This, however, assumes that the relationship between the secondary pressures and the quantities of steam they control can be represented by a straight line and, as a matter of fact, this can be attained in practice by fitting throttle collars to the seats of the valves.

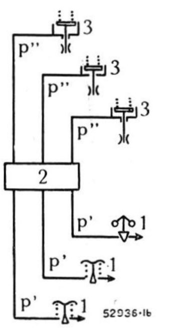


Fig. 2. — Fundamental arrangement of the new multiple governing system.

1 are the governors,
2 is the pressure transformer,
3 are the governing organs,
p' are the pressures regulated by the governors and p'' are the pressures regulated by the pressure transformer.

The measures to be taken in order to satisfy the second and third regulating conditions always depend on a limitation of the primary pressure. This is carried out by the differential pistons through the additional control edges A_1 and A_2 which allow more oil to escape from the primary pressure system or suppress existing oil escape apertures (Fig. 3). To carry out these duties the differential pistons move away from their normal regulating position and slide upwards or downwards until one of the additional edges comes into action. In order that this displacement should take place at exactly the right moment it is arranged that the limit of the regulating range of one set of valves coincides with the limit of the corresponding secondary pressure and this is nearly always practicable by suitable choice of the valve springs. Thus, the secondary pressures attain their maximum value, which coincides with the governing oil pump pressure, at the instant the valves they control are full open. Obviously, this applies to the lower limit as well. Every time such conditions occur, one of the secondary pressures cannot increase or decrease further, thus losing its capacity to maintain the differential piston in equilibrium. The latter then moves away from its normal position and takes over the duty of limiting one or other of the primary pressures. We will take as example the most frequent cases in order to explain how these various measures work out in practice.

Extraction governing.— Fig. 4 shows the simplified diagram. The operating magnitudes in function here are the output and the extraction quantity. The regulated magnitudes related thereto are the speed and the extraction pressure. The primary pressures are p_n and p_e, the secondary pressures are p_f and $p_ü$. According to the first governing requirement the extraction pressure should not be affected by a variation in the load delivered by the turbine and the speed of the turbine should not be affected by a variation in the quantity of steam extracted. Now this condition is satisfied if the steam inlet valve and the extraction pressure regulating valve open and close in such a manner that when a variation in load occurs the amount of steam flowing through varies by the same amounts and that, when a variation in quantity of steam extracted occurs, the load variations in the high pressure and low pressure parts compensate one another. According to this and to what is shown in Fig. 5, the pressure p_n which determines the output is made to act on both differential pistons, in the upward direction and the pressure p_e which determines the extraction in upward direction in one differential piston and in downward direction in the other. In the case of a variation in load, both sets of valves of the pressure transformer move in the same direction, and in the case of a variation in extraction quantity they move in contrary directions. In order that this should give results exactly according to the first governing conditions, it is necessary that the sections of the differential pistons be suitable chosen.

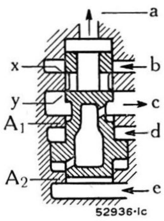


Fig. 3. — One of the regulating slide valves of the pressure transformer.

x and y are the regulating edges for inflow and outflow of oil. A_1 and A_2 are edges to limit the primary pressure.
a. Secondary pressure.
b. Pump pressure.
c. Outflow.
d. 1st primary pressure.
e. 2nd primary pressure.

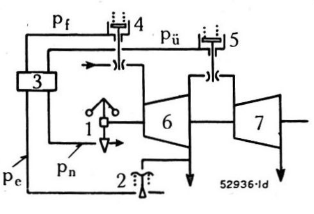


Fig. 4. — Simplified diagram of the governing system on an extraction turbine.

1. Speed governor.
2. Extraction-steam pressure regulator.
3. Pressure transformer.
4. Inlet valve.
5. Extraction-pressure regulating valve.
6. High-pressure stage of turbine.
7. Low-pressure stage of turbine.

p_{ne}, p_e. Primary pressures.
p_f, $p_ü$. Secondary pressures.

Closer details of the pressure transformer can be taken from Fig. 6 which gives the complete governing diagram of an extraction turbine. There are four additional edges A_1—A_4 added to the differential pistons which take care of correct governing when limit conditions are reached. An example should show how they work. Assuming that the steam inlet valves are full open, the high-pressure blading of the turbine fully

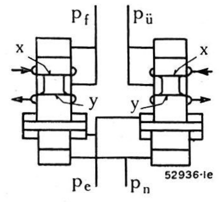


Fig. 5. — Simplified section of the pressure transformer of the extraction governing system.

x and y are the inlet and outlet flow regulating edges, by means of which the secondary pressures p_f and $p_ü$ are regulated. p_e and p_n are the primary pressures.

loaded, then still heavier loading can only be carried by the low-pressure part at the cost of a reduction in the quantity of steam extracted. From this moment, either the speed must drop or the extraction pressure must drop. In most cases it is more essential to keep up the speed as an accidental drop in the quantity of steam extracted can be covered by a reduction valve. When, therefore, this extreme condition occurs, the

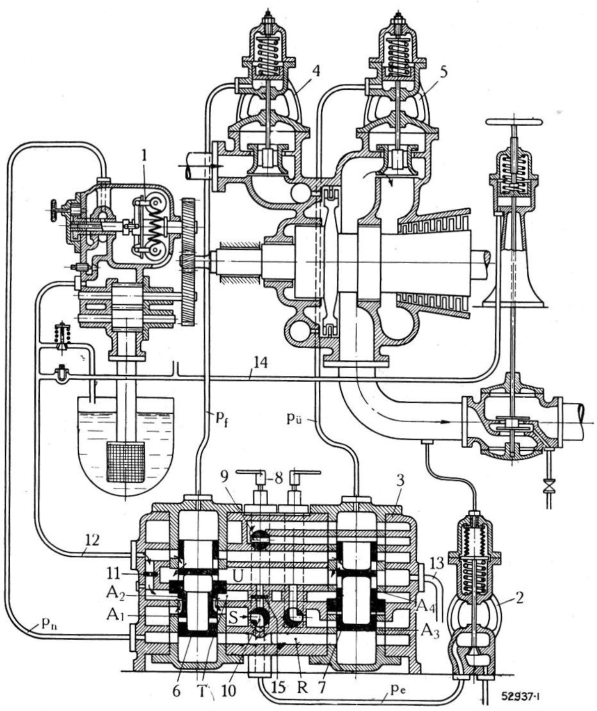


Fig. 6. — Complete diagram of extraction regulation.

With the exception of the pressure transformer, all the parts are of standard design.

1. Speed governor.
2. Extraction-pressure regulator.
3. Pressure transformer.
4. Inlet valve.
5. Extraction-pressure regulating valve.

6—7. Differential pistons.

8. Handles to change over to operation without extraction.

9—10. Cocks for changing-over.

11. Inflow diaphragm for extraction pressure regulator.
12. Inflow of oil under pressure to pressure transformer.
13. Oil outlet.
14. Stop-valve system.
15. Diaphragm to adjust p_e when extraction is cut off.

A_1 to A_4. Edges to limit the primary pressure.

following processes take place. Secondary pressure p_f attains its maximum with the full opening of the inlet valves, the governing piston 6 moves up and its edge A_1 unmasks an oil escape opening leading from chambers S and T into chamber U; this causes a limitation of pressure p_e. In this state, the value to which this pressure is limited is not fixed but varies according to p_n so that it can be said that piston 6 is no longer governing pressure p_f but pressure p_e. Pressure p_e now takes over the recall of the governing piston in place of p_f, the new position of piston 6 is thus a new position of equilibrium. The latter will be maintained as long as the limit condition lasts. The result is that the extraction-pressure regulating valves do not close further than is compatible with the maintenance of the turbine output so that, in reality, the speed is maintained at the cost of a drop in extraction pressure. The change of the differential pistons from one position to another is quite automatic and smooth and cannot be perceived from outside. The pressure regulator behaves quite analogously in the other limit states and thus fulfils the second governing condition.

The same example should illustrate as well that these actions of the governing apparatus are in conformity with the third governing condition. In the present case, it is required that the speed drop characteristic of the turbine should not be influenced by the process. Therefore, a given variation of p_n should always result in the same variation in output, this whether the inlet valves are partially or quite open. In the first case, a variation in load is shared by the high-pressure and the low-pressure parts, in the second case the high-pressure part is already fully loaded and cannot participate in an increase of the load, which has to be born by the low-pressure part alone. This is the case, as a matter of fact, because as soon as the inlet valves are fully opened, a change in the primary pressure p_n to $p_ü$ has a double effect, once it acts directly as in a normal governing process and again through the other governing valve by a corresponding change of p_e, which also reacts on $p_ü$. Detailed examination of the action, for which we have not the space here, shows that the second change of $p_ü$ is just of that magnitude to produce that increase in the output of the low-pressure part which covers the lack of increased power from the high pressure part.

This is no accidental result but an expression of the strict law on which the governing system is based. Therefore, for all other cases of limit governing the behaviour of the apparatus is analogous and the third governing condition is, thus, satisfied.

It should be mentioned that the pressure transformer can be adjusted for operation without extraction. To this end, the cocks 9 and 10 are rotated by handle 8, which frees the extraction steam regulating valves from the influence of the governing valve 7 and opens them full, oil under pressure being supplied under their respective pistons, on the other hand the extraction pressure regulator is cut off from the pressure transformer. This change-over, as well, is accomplished, practically, with perfect smoothness as, after the change-over, the diaphragm 15 adjusts pressure p_e to the same value as the pressure regulator did before.

Extraction two-pressure governing. If steam is not only extracted at the extraction point but also supplied, from time to time, to the turbine, the latter is classed as an extraction two-pressure turbine. To a certain extent its governing is that of a modified extraction turbine and is based on the same principles. However, the relationship between p_e and the quantity of steam extracted according to Fig. 7 line a, must be replaced by one according to line b, which can be accomplished by putting in two springs to balance the pressure p_e^*. This is already provided for in the design of the pressure transformer for extraction governing and the transformer can be used for both types of governing. It is, therefore, only built to one size. Its outward appearance is shown in Fig. 8 and its simple lines fit into those of the turbine in pleasing manner. The pressure transformer is located at the side of the turbine on a bedplate. As it is the only new element in the governing equipment, any existing multiple governing system can be transformed into the one described here simply by the addition of a pressure transformer. The new governing system has already been built and fitted to steam turbines and has fulfilled every expectation.

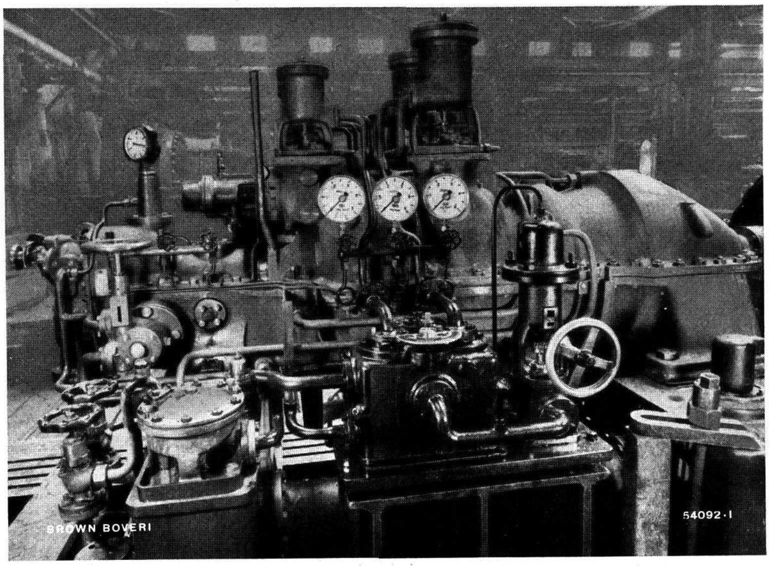


Fig. 8. — **View of the pressure transformer and extraction pressure regulator.**

Multiple governing with three conditions to be fulfilled. — The principle of the differential piston can be used advantageously for all other multiple governing systems. The number of controlling valves simply increases with the number of operating conditions it is necessary to satisfy. Despite this, the layout remains very simple, as Fig. 9 shows. When compared thereto, governing by rods for three governing conditions can hardly be considered as an acceptable solution, the great superiority of hydraulic governing as compared to governing by rods being very marked.

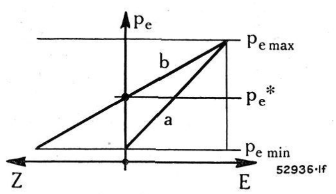


Fig. 7. — **Relationship between quantity of steam and primary pressure p_e.**

a. In an extraction turbine.
b. In an extraction, two-pressure turbine.
Z. Additional quantity on low-pressure side.
E. Extraction quantity.

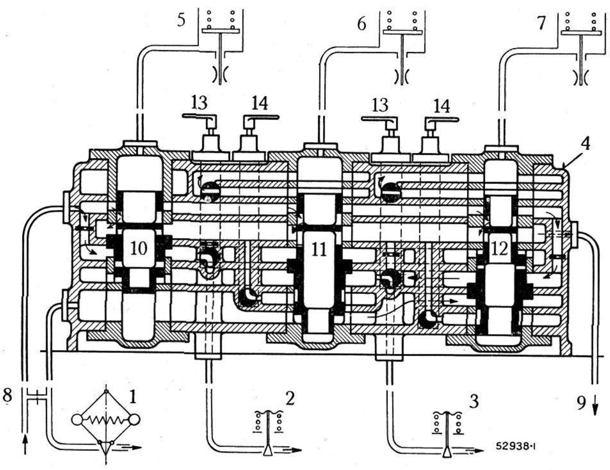


Fig. 9. — **View of a double extraction turbine.**

1. Speed governor.
2. Regulator for a 1st extraction pressure.
3. Regulator for a 2nd extraction pressure.
4. Pressure transformer.
5. Inlet valve.
6. 1st extraction pressure regulating valve.
7. 2nd extraction pressure regulating valve.
8. Oil from pump.
9. Oil escape.
10. Differential piston.
11. Differential piston.
12. Differential piston.
13. Handle to adjust operation with or without extraction.
14. Handle to adjust operation with or without limitation of extraction pressure.

Among other plants, the new governing system is to be fitted to two double extraction turbines each of 30,000 kW and to an extraction and two-pressure turbine of 12,000 kW.

(MS 805) *A. Leyer. (Mo.)*

SOME RECENT INVESTIGATIONS INTO SEGMENTAL THRUST BEARINGS.

Decimal index 621.822.2

A description of tests carried out on segmental thrust bearings appeared in The Brown Boveri Review of July/August 1933. Here, the thrust collar was made of glass in order to render visible the flow of lubricant between thrust collar and the segments. The observations carried out during these tests led to a series of further tests which produced extremely interesting results. We only describe the more important of these, here; they were made in order to collect data on the load-carrying capacity of the segments.

BEFORE tests were made with a glass thrust collar, it was thought desirable to lodge as big a surface as possible in a given thrust bearing. This, however, led to narrow gaps between the segments and to insufficient lubrication which, in its turn, reduced the load-carrying capacity of the segments. The question then arose of how big the gap between segments should be in order to impart to the segments the biggest load-carrying capacity.

This problem was solved by making loading tests on thrust bearings with varying numbers of segments. A thrust collar was mounted at each end of a shaft and a thrust bearing pressed against each block by means of a hydraulic press. In all tests, the shaft was rotated at 3000 r. p. m. The pressure was increased until a breakdown occurred. In accordance with what is seen in the diagram of Fig. 1, tests were carried-out in the sequence of 10, 8, 6, 4, 3, 2 and 1 segments.

To begin with, it was necessary to make certain that the pressure was evenly distributed between all the segments. Fig. 2 shows how the segments are supported.

All the segments are of equal height and rest on a ring R; they can tilt round the edge A and are maintained in position by a pin. The ring has a certain flexibility and it is supported under each segment, by a small copper cone K. The dimensions of the cone are determined by means of the hydraulic press in order that it may be compressed by some millimetres under the load it is expected to carry. In this way as uniform a distribution as possible of the load among the different segments is obtained. In order to eliminate surface irregularities, every segment which had worn down at all, was replaced. Fig. 3 gives the results obtained.

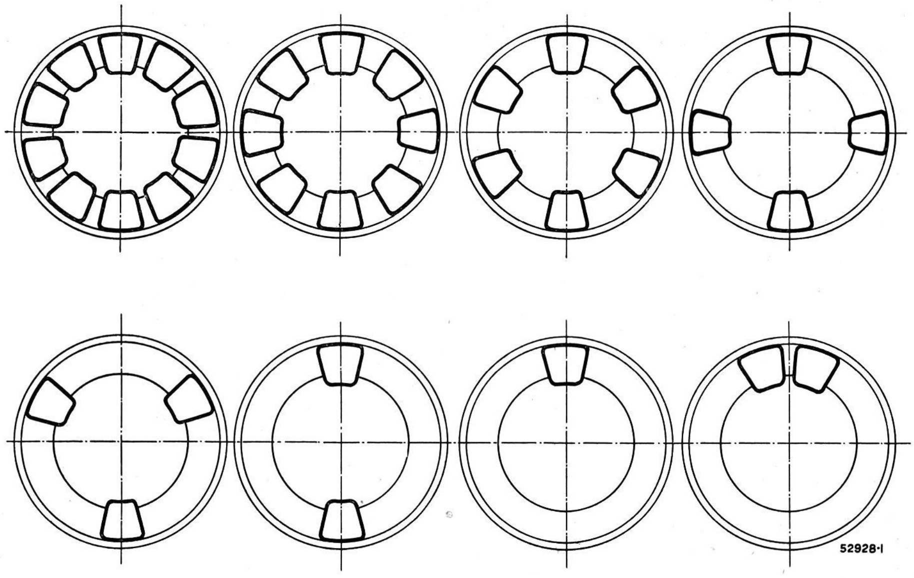


Fig. 1. — Arrangement of the segments for the different tests.

The number of segments is plotted as abscissae axis and the load of each segment which began to wear down the surface is plotted above it (curve 1). This curve shows a *peak value for 6 segments.* The gap between segments is, here, about 40 % of the carrying surface. The arrangement with 6 segments instead of 10 gives an increase in load-carrying capacity of the bearing of 50 %.

If the load on the bearing is divided by the carrying surface, we obtain the specific load at which the segments begin to be worn away. In the case of a single segment, this value is somewhat more than 400 kg/cm² and it drops to 100 kg/cm² when there are 10 segments. If a line is drawn to connect these points, it is found to be nearly straight (curve 2). Thus, it is seen that the load-carrying capacity of a single segment alone is about four times greater than it is when there are 10 segments.

To conclude, a test was made in which two segments were mounted in the same bearing beside one another, but separated by the same gap as if 10 segments had been mounted. The result was point B in Fig. 3. This corresponds to 325 kg/cm² while it should have been 265 kg/cm² if the first segment had behaved as a single one and the second segment as one of 10 segments. It is probable that the oil flowing to the second segment is colder than when there are 10 segments, thus explaining the higher load-carrying capacity.

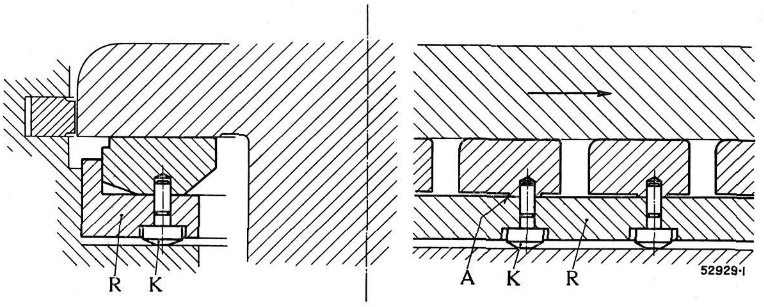


Fig. 2. — Support of the segments.
A. Supporting edge of segment. K. Copper cone. R. Supporting ring.

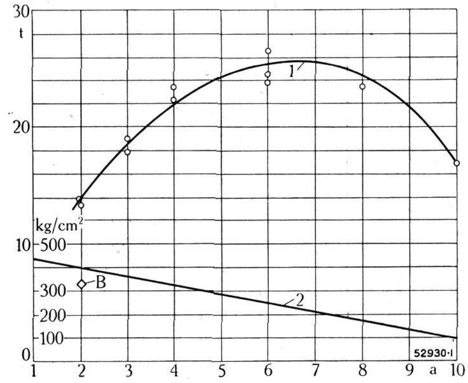


Fig. 3. — Load-carrying capacity of a thrust bearing.
Abscissae :— a. Number of segments.
Ordinates :—
Curve 1 : Total pressure in t on the segments.
Curve 2 : Specific pressure on the segments in kg/cm².

For some years now the results of these investigations have been put to good use in Brown Boveri thrust bearings, the successful result being an increase of 50 % in operating reliability.

(MS 806) *Dr. J. v. Freudenreich. (Mo.)*

THE DEVELOPMENT OF TURBO-GENERATORS OF BROWN BOVERI DESIGN.

Decimal index 621.313.322-81 (091)

A chronological summary is given here of those constructional features which are milestones in the progress of turbo-generator design. The actual state of development is illustrated by a description of the characteristic features of a 56,500-kVA generator. Some present problems inherent to the building of big generators are touched on and indications are given of the lines along which future development probably lies.

THE design of the rotors of turbo-generators, as laid down by C.E.L. Brown in 1901, formed a basis which has suffered no revolutionary changes, up till to-day. On the other hand, based on the fundamental design in question, there have been exceptional developments which marked the fight for the mastery of the forces in play and the search for materials best suited to that end. These developments are due to a number of enterprising brains. The first test reports carry the names of such men as Aichele, Nizzola, Marguerre, Vanotti, Fr. Sauter, etc. who have since gained leading positions in the electrical industry. The posts they were subsequently called on to fill and the honours bestowed on them probably gave them no keener pleasure than did their early successes achieved on the test bed and at acceptance tests. The purely designing side of turbo-generator construction was certainly no less colourful; as regards our firm, E. Hunziker had this in his hands for more than a normal span of life. He was a faithful defender of the principles laid down by Sydney Brown for the design of generators.

The variety of designs was far greater in early days than it was in the later stages of development. Thus we built laminated rotors with semi-closed slots and very soon afterwards massive rotors, others in which plates were mounted side by side on a shaft, then again massive rotors with axial cooling and arrived finally at a built-up rotor bolted together, which freed us from the dangers of internal stresses and defective forgings. The problem of critical speeds is bound up to that of rotor design. Here it was a question of shaking off the restrictions of too strongly idealistic theories, which still form a stumbling block to a part of the technical world. The rotor end caps were composed of cast bronze, the material for which could only be obtained from England, at first. Replacement thereof by magnetic steel led to inadmissible losses. Then the classically perfect bronze produced by the Lauchertal works helped to overcome the difficulties until the increase in the dimensions made it necessary to resort again to magnetic end caps which were combined with a bronze intermediate ring to reduce its disadvantages. When, finally, non-magnetic steel end caps became available, the difficulties seemed to have been overcome for a long time to come. Despite this, there were many to be found who expressed warnings against their use, although, to-day, the most stubborn opponent has been won over by their obvious advantages.

At the beginning, the rotor winding was laid in two layers side by side in each slot. The change over to one layer per slot was a big step forward as regards service reliability. This progress was materially helped by the readiness of clients to adapt the excitation voltage to suitable values; this was accomplished only thanks the high standard of reliability of our built-on exciters. From this period onward it was not necessary to tie oneself down, as regards excitation voltages and dimensions of conductors, to stand-by sources of excitation current but could choose what best suited the generator design.

As the temperatures considered admissible in generators increased, the first insulation used, composed of presspahn and cotton, had to be replaced by other materials of higher heat-resisting qualities, such as mica compositions, which, however, were not always satisfactory from the point of view of mechanical resistance. Very excellent results were obtained, however, with certain compositions developed by us at a somewhat later date and composed of asbestos fabric and synthetic resin. Quite recently, we have been considering using glass fabric which has exceptionally good thermal and mechanical properties.

The stator winding had unpleasant surprises in store for us, especially as regards heavy-current generators having one bar per slot, on account of the high additional losses. At first, experiments were made to reduce "Field" losses with bars of round section, then with oval ones. The first perfect solution, however, was contributed by our collaborator L. Roebel and consisted of artificial bars built up of a number of transposed conductors; to-day, this solution is still considered by us as being a perfect one. It is interesting that the former round bar is being used again in modern high-voltage generators, but it is made up of a subdivided conductor, according to what we know to-day (Fig. 1).

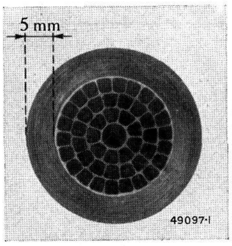


Fig. 1. — Section of a stator-winding bar for a 22,000-V generator.

The wall thickness of the insulation of a 22,000-V generator with a round-bar winding is only 5 mm.

In dealing with windings of lower mechanical rigidity and having several conductors per slot, we were surprised to observe the effects of the forces created during short circuits which had been hardly noticed in the case of multiple-pole machines. This led to a development of the supporting pieces of the winding until the desired mechanical strength was acquired (Fig. 2). The laying of the stator-end windings, which was first made parallel to the axis of the machine, was modified to an inclined position. The basket or ball winding thus attained was used by us for many years, to be finally abandoned in favour of an end winding laid parallel to the pressure plate which allows of greater precision in bending, insulating and clamping the winding. Our own special shape of coil brings out the advantages of this kind of winding to the full. Every single element of the winding can be interchanged for another, even with semi-closed slots, the shortening of the winding pitch can be made with regard to the minimum of higher harmonics and is not determined by considerations of difficulties in putting in the winding, the stray reactance remains big enough and the ventilation draught is uniformly distributed. As the axial length of stators kept on getting bigger and bigger, it was found that fissures in the stator-winding insulation began to appear. The insulating mantle pressed on round the bar was damaged by swelling up of the insulation into the slits left for cooling, combined with unavoidable displacements due to heat expansion. This

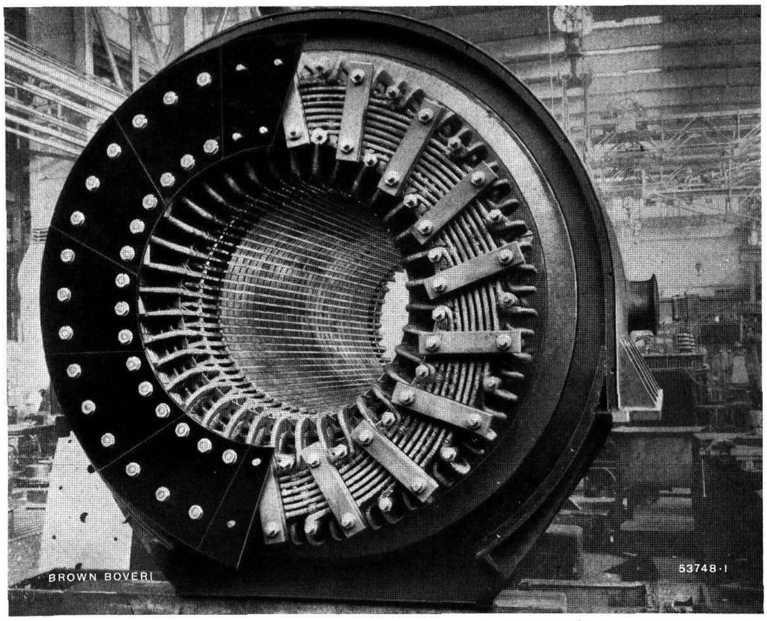


Fig. 2. — Stator of a 37,500-kVA turbo-generator with part of the shield securing the winding removed.

The arrangement of the stator-end windings parallel to the pressure plates leads to simple shapes for the coils, which are especially suitable for the reliable construction of the coils themselves and for supporting them in position in the generator.

difficulty was overcome by means of a suitable design of the distancing pieces in the slits so that there was now a continuous wall to the slot which presented no obstacle to sliding movements of the bar caused by expansions and contractions. This innovation was to our knowledge applied for the first time on a Brown Boveri generator. Fig. 3 shows how these separation pieces or fingers meet the aerodynamical requirements for air inflow and outflow, in the case of semi-closed slots, which are practically always used by us.

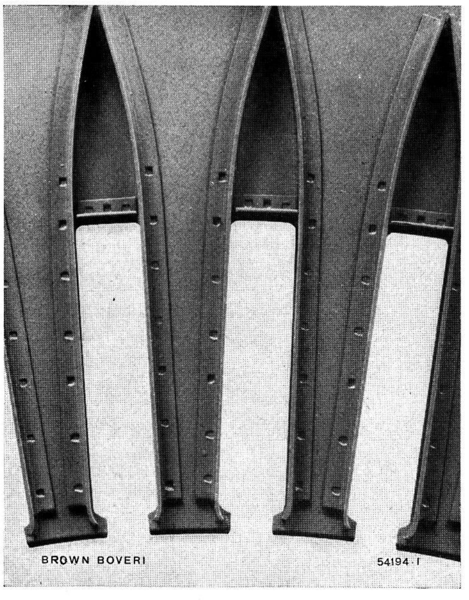


Fig. 3. — Air-guidance pieces in the stator laminations of a turbo-generator.
To reduce losses in pressure the guidance of the air flow must be carefully studied.

Happily, we met with none of the adverse phenomena in the laminated body of the turbo-generator, of which much has been written. Paper insulation of the laminations, as carried out by us, has given most satisfactory results. There is no deformation of the laminations as occurs when they are being enamelled and the insulation is very effective. By subsequent treatment with a special varnish, the films of air between laminations can be permanently eliminated, the heat conductivity perpendicularly to the laminations increased and an effective protection against external damage attained.

A difficult problem, which we solved in most satisfactory manner, was that of the assembling bolts passing through the laminations; we managed to eliminate them altogether by a suitable design of the pressure plates at each side and of the generator housing. The damping winding in the stator formed of the keys on the inside periphery of the stator housing for securing the laminations and which are made of material of high conductivity forms a very effective protection against damage due to electrical causes. Further, we had no trouble as regards mechanical vibrations in the laminated body. The heavy cast housing which was only succeeded by the welded steel housing, relatively recently, thanks to the excellence of our castings, were sufficiently insensitive to vibrations. The same applied to the welded steel housings which, as a sort of heritage from the days of cast housings, are made sufficiently massive.

It meant a welcome simplification to the construction of the stator when the decision was taken to make the stators in one piece, thus putting the requirements of design before those of freighting and erection. When necessary, the stators were laminated and wound on site. Before this, 2-, 3- and 4-part stators were encountered in which, however, the big section of the laminated body gave rise to difficulties as regards the gaps between the sections.

The directly built-in cooler, which appeared in recent years, was an innovation we did not adopt and we had no difficulty in justifying our rejection thereof.

As to the lines of our housing, we made great efforts, at an earlier period, to conciliate the taste of the time which was much influenced by what was being produced in other fields. We are far freer to-day in this respect. The technically-trained eye is now satisfied if it can acknowledge the suitability of a design to the purpose to be fulfilled and a workmanlike, consciencious construction.

The duty of the housing, apart from being the support of the laminations, is to distribute the cooling air through various ducts and chambers. A very simple construction of the generator housing was evolved in conjunction with separate ventilation. This consists of a series of supporting rings perpendicular to the axis of the generator; these are connected together on their inner periphery by longitudinal bars and on their outer periphery by a solid steel mantle. The resulting subdivisions of the space form the channels for the air inflow and outflow.

The tendency towards greatest simplicity was also manifest in the cooling of the rotor. We learnt to limit cooling to that of the outer surface but this relatively simple cooling method could only be put into practice after the manufacturing process of all the parts which participate in the carrying off of the heat had been sufficiently developed.

As regards cooling, turbo-generators held a privileged position from the first. At a time when forced ventilation applied to other classes of machines was looked upon as being a sign of a certain inferiority or an admission that the machine was overloaded, built-in fans were considered the obvious solution in turbo-generators. If, at first, it was noise that forced designers to give serious attention to the cooling problem, the increasing losses due to ventilation, as the circumferential speed got bigger, contributed considerably towards a serious study of the cooling problem. In other fields of electrical machinery construction where this need was not so great, the development was slower. The laying down of hot and cold air ducts, the utilization of filters and finally the adoption of closed-circuit cooling had long been standard solutions in turbo-generator plants when they were still exceptions in other big machines. In designing fans, we always had available the experience which our experts had acquired in the blower and compressor field. When the axial type of fan, the development of which is due to the design of aeroplanes, proved the most advantageous as regards efficiency, the former centrifugal type fans were given up although, constructively, they had proved excellent and well, even better, able to supply the pressure needed (Fig. 4).

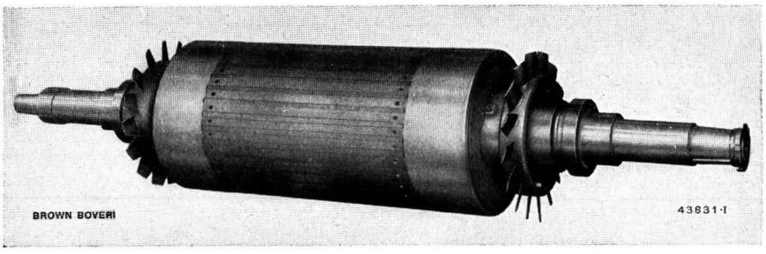


Fig. 4. — Rotor of a medium-size turbo-generator with axial-type fans at both ends.

The preceding paragraphs should show what difficulties were presented by the further development of various parts. Many of our solutions to surmount such difficulties developed into epoch-making contributions to turbo-generator design. Once again it was confirmed that, finally, it is always the — in widest sense — most economical machine which wins through. Designs which owe their popularity to slogans of a period are generally abandoned after a short time. Auxiliary duties imposed on the generator at the expense of its economical operation or reliability are short-lived and there are usually other means found to carry out the duties in question at lower cost and more completely.

The long life which characterized turbo-generators had the unfortunate effect of making owners tolerate plants which were no longer economical to operate, the owner being loth to scrap perfectly good machinery, this despite the fact that one per cent better efficiency would soon cover the expense of a new generator.

Of 3200 turbo-generators with a total output of 16 million kVA built by the Brown Boveri concern, very few indeed have been scrapped and then, generally, only when their frequency, phase system or voltage no longer harmonized with new operating conditions.

All the experience gained and the number of designs brought out and studies made in the course of forty years of development led to machines which, despite increasing requirements, were in the vanguard of their class as regards efficiency, utilization of material, saving of space as well reliability in operation.

The description of a generator of 56,500 kVA, p. f. = 0·8, 11,000 V, 3000 r. p. m., 50 cycles, is given here to illustrate what we are in a position to perform, to-day (Fig. 5).

With a total weight of 80·5 t of which 21 t are for the rotor, we get about 1·4 kg per kVA produced. The short-circuit current is within the limits recommended by standard rules, namely 15 times the rated current. The excitation requirements at the rated load are 120 kW. The amount of air needed is 20 m^3/s and is delivered by two separately mounted fans which produce a pressure of 300 mm WG. The two motors driving the fans each take 55 kW. The closed-circuit cooler is dimensioned for 600 kW at 30 mm WG pressure drop. The following table gives the efficiency, the bracket figures being valid for partial loads when only one fan is running:—

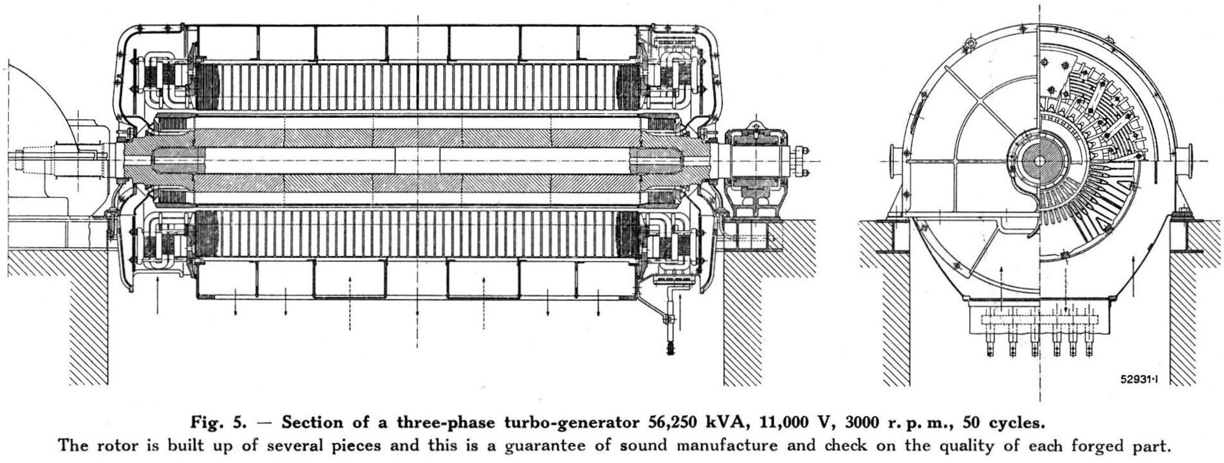


Fig. 5. — Section of a three-phase turbo-generator 56,250 kVA, 11,000 V, 3000 r. p. m., 50 cycles.
The rotor is built up of several pieces and this is a guarantee of sound manufacture and check on the quality of each forged part.

Load in kW	at p. f. = 1		at. p. f. = 0·8	
56,500	98·7	—	—	—
45,000	96·6	(98·75)	98·25	—
33,750	98·4	(98·6)	98·1	(98·30)
22,500	97·8	(98·1)	—	(97·9)

In order to attain as high an efficiency as possible, efforts will be made to eliminate the additional losses above all others. The losses encountered in the stator-end winding connection spaces are diminished by using non-magnetic material on all parts subjected to the strongest fields. The inside border of the pressure plates and the fingers holding the laminations of the stator teeth are made of bronze, the parts for clamping the windings are of non-magnetic steel, the very high strength of which give the necessary guarantee of rigidity in face of the short-circuit forces. Thick-walled aluminium shields contribute as well to reduce losses. Metallic parts to support and stiffen are to be avoided as far as possible in close proximity to the winding; compositions of synthetic resin should be used for the purpose.

The additional losses due to the stator current created in the massive parts amount to only 0·2 % of the total kVA figure for this 56,500-kVA generator and despite the heavy current of 750 A per cm of internal periphery. As this percentage drops with the square of the load, the curve of additional losses at partial loads drop steeply.

The additional losses inherent to the magnetic circuit in the stator laminations and on the surface of the rotor are low thanks to the semi-closed slots. All distancing pieces in the cooling slits in the range of strongly saturated zones are of non-magnetic material. On the other hand no attempt has been made to reduce these losses by lowering the thickness of the laminations to a value not mechanically recommendable or by using mechanically unsuitable alloys.

In order to reduce the power required to supply the necessary ventilation, while maintaining velocities of air flow sufficient for cooling purposes and keeping the machine clean, aero-dynamic principles are applied in designing the principle air ducts. Here also the semi-closed slots proved advantageous, as there is no restriction of section due to slot keys and the broad tooth top facilitates imparting a nozzle shape to the distancing pieces. Proper guidance of the air flow prevents eddies forming at the roots of the teeth. The greatest saving is achieved by applying the system of separate cooling, by means of which the fan losses and the heat generated by the compression of the air in the fans are kept away from the interior of the generator. This allows of reducing the amount of cooling air necessary, and the separate fans can be built for a higher efficiency as their design is not restricted as to available space. The difference in temperature between hot air and cooling water can be increased to the advantage of the cooler by the amount corresponding to the power necessary to drive the fans, this while maintaining the usual temperature rise of the air allowed for in the generator. The air is carried to the stator in several partial streams which are easy to control. As constructive advantages we would mention:— considerable shortening of the overall length, simplification of end shields and of housing proper, elimination of rotating fan blades in close proximity to the end connections of the winding of the stator, the span of which blades exceeds the diameter of the rotor, in big units, thus complicating erection.

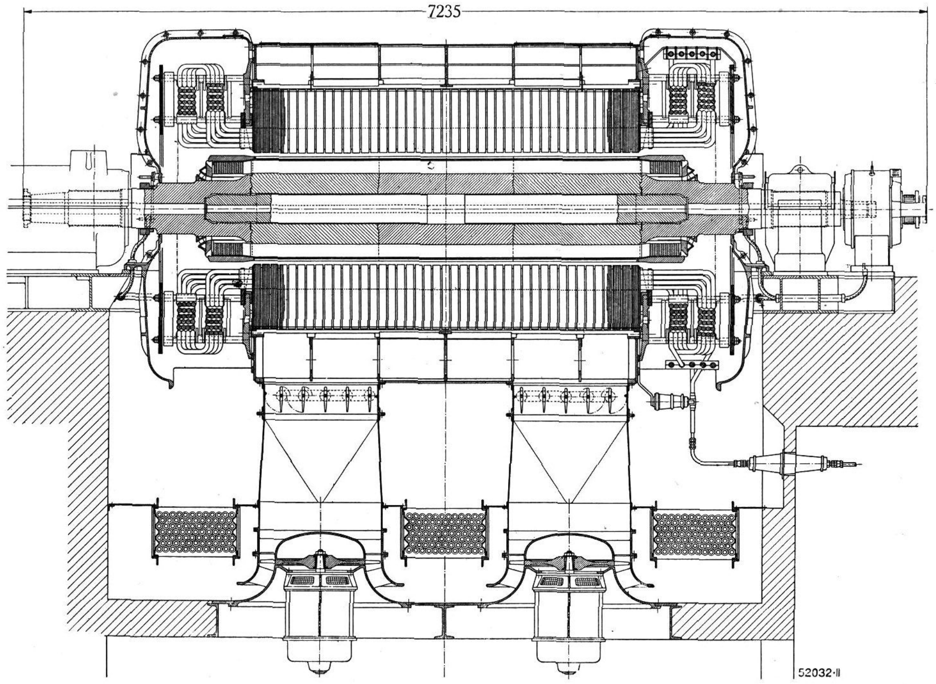


Fig. 6. — Section of a three-phase turbo-generator 31,250 kVA. 22,000 V, 3000 r. p. m., 50 cycles, with cooling fans mounted separately.

It is easy to find room for the separate fans in a restricted space. These separate fans simplify the design of the generator housing and shields, they allow of shortening the overall length of the generator and they improve its efficiency.

As is shown in Fig. 6, the separately mounted fans can be combined with the generator in a compact manner. The subdivision of the cooling duty between several fans, means a saving in space and a greater guarantee against a breakdown of the cooling; at partial loads this layout permits of reducing the power absorbed for cooling purposes, as it is possible to stop one fan altogether.

In some countries, separate cooling was looked on with disfavour, up till lately, on account of supposed complications which it entailed. It is a remarkable fact that hydrogen cooling was looked on with rather more favour. But the same disadvantages can be argued against the latter kind of cooling, as long as the advantages accrueing therefrom do not outweigh them.

The way in which the rotor is cooled influences the amount of power expended to overcome air friction, which represents a considerable portion of the losses. Simple cooling of the outside surface is the most advantageous in this respect, because of the smooth surface of the rotor. It causes a minimum of disturbance to the streams of air crossing the air gap from the stator, further the air flowing in from both sides, the cooling influence of which is very effective, is carried off smoothly through the slits in the stator housing and is not affected by jets of air thrown off by the rotor. There being no air ducts in the body of the rotor, there is more room left for the copper of the rotor winding. Thanks to the large amount of copper used, the fraction of the losses caused by the excitation is small. Other advantages are lighter loading of the slip-rings and the small instantaneous-reaction exciter. With this design, there is no soiling or clogging of inaccessible parts which is otherwise to be feared despite closed-circulation cooling. Apart from the advantages mentioned, cooling of the external surface only is very advantageous as regards lodging the coils in the slots and permits or facilitates the building up of rotors in several parts which is a better design, as regards safety, than rotors made out of one piece, when dealing with big machines.

The winding is protected against any deformation on the whole length of the iron core by the walls of the slots which are continuous on the whole length. As the difference of temperature between iron and copper is low, the much feared differences in elongation due to expansion are kept within reasonable limits. The coils themselves which are baked under high pressure in the slots have no tendency towards displacement in the coil ends and the distancing pieces can be made small and the surfaces of the coils kept free for cooling. The scavenging of the intermediate air spaces is taken care of by the natural-cooling action, which is very effective in the field subjected to the strong centrifugal forces set up, while there are carefully-designed fans at the air entries to look after sufficient renewal of the air. The hot air is ejected into the space of the stator shields through slits in the cap plates. This design prevents the heat due to the losses at the two coil heads being led directly into the air gap between stator and rotor which is already considerably utilized; holes in the end-cap rings, which weaken these pieces, are also avoided. The section of the air inflow to the cap plates is ample thanks to a new locking system which takes up little space and has proved a simpler and more easily-loosened device than the holding nut.

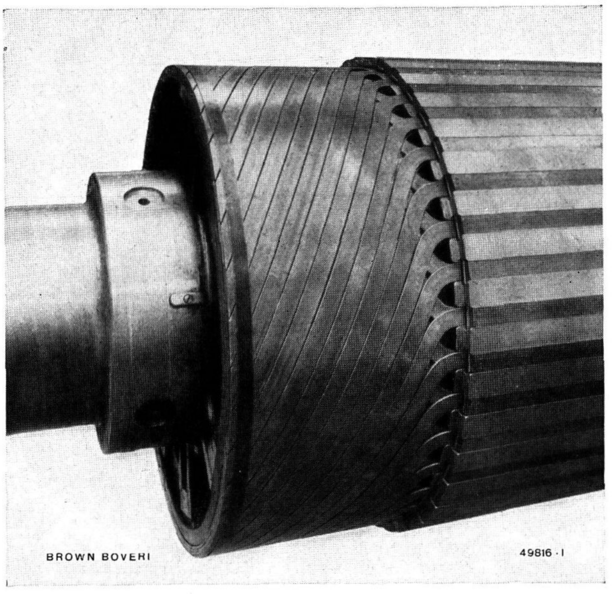


Fig. 7. — Damping winding in the rotor of a single-phase turbo-generator.
The subdivision of the damping winding into independent elements gives the necessary play and facilitates putting the winding in and taking it out.

Usually, damping windings are not used because no need for them has arisen in our turbo-generators. However, when dealing with exceptionally unequal loads of long duration or in the case of single-phase generators, the damping winding cannot be dispensed with. The design shown in Fig. 7 has met the severest requirements made on damping windings.

The most important special feature of Brown Boveri design, incorporated in medium and big units, is the built-up rotor.

For over a decade, discussions have been going on as to the best processes to apply in forging and alloying big forged pieces in order to obtain faultless material devoid of inner stresses. These discussions and the methods employed to check the qualities of the forged pieces, which methods are always being improved and were, in part, very complicated, show that to-day the manufacture of such parts is still a difficult business and one characterized by much scrapping of defective forgings.

The relatively small forged pieces required for the generator described here could be produced in medium sized iron works such as we have in Switzerland and this in high-grade quality, with a yield point of up to 60 kg/mm^2, a tensile strength of 75—80 kg/mm^2, 22—20 % elongation and an impact value (notch-bar test) of 15—12 kg/cm^2, that is to say with far higher values than are necessary to satisfy the usual factors of safety. The pressure exercised through the tension bolt on the drums by the two shaft ends which

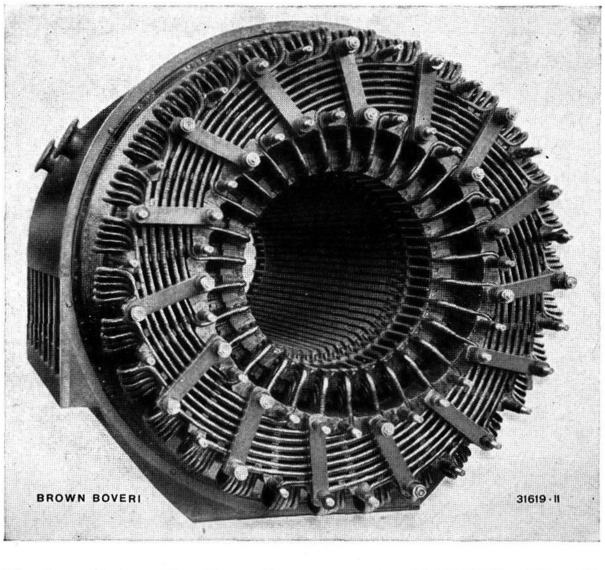


Fig. 8. — Stator of a three-phase generator 31,250 kVA, 36,000 V, 3000 r. p. m., 50 cycles.

The stator winding for 36 kV is as simple to build up as that of a low-voltage generator.

function as nuts exceeds 1000 t and imparts to the rotor the properties of a solid piece.

In looking out for new development possibilities, very promising lines have been opened up. One way to improve the overall economy of the plant which holds out good prospects, but which has only been used so far in Great Britain and the Dominions, is the construction of high-voltage generators. We began investigations in this field, ourselves, when the first signs of a demand for such units was perceived (Fig. 8). With these high-voltage generators, it is possible to save nearly the total transformer losses and a good part of the material used to build the transformer and switchgear. The need of a machine of this type was all the more obvious as the manufacturing methods of insulating material of first-class mechanical and electrical qualities, which had become known by their utilization in machines built for extreme conditions, gradually became common property.

Obviously these different insulations are also employed to build standard-voltage generators, as well, where they allow of better utilization of the material.

The progress made in the manufacture of dynamo laminations will probably soon be applied to the laminations used in generators and will lead to lower losses as, indeed, has been the case for transformers since many years. If this development has been retarded, this is due to the greater requirements made, in generator construction, on the mechanical strength of the laminations and on their resistance to warping at stamping.

The additional losses in the pressure plates can be practically eliminated, at least theoretically, by the so-termed "absorption sheets", however this method can only be applied, for the time being, to the simplest shapes of windings.

Every reduction in the losses is accompanied by a drop in the power outlay to drive the fans. A reduction on losses is, further, attainable when separate ventilation is used, where axial-type fans allow of attaining, in some cases, efficiencies of 80—85 %. Increasing experience in building big collecting ducts should make possible a reduction of the not inconsiderable losses in these channels.

In the case of air-cooled generators, there is no reason to consider a 99 % efficiency as belonging to the realm of Utopia. Despite this, the further gains which may be attained when using hydrogen cooling will certainly by made use of in plants where the other conditions for this type of cooling are favourable. It is, however, questionable, in view of the high efficiency attained by air cooling, whether it is worth while changing over to hydrogen cooling. Hydrogen cooling, for the case under consideration of the 56,500-kVA generator, with two fans running would mean a saving of a total of 200 kW in air friction and fan losses and one of 150 kW with one fan running.

If a big number of similar machines were put up in a power station, the attendance necessary for the hydrogen cooling equipment would have little importance. In medium-sized stations the situation is different. If frequently happens here that the amount of stand-by power is so small that overhauling work must be reduced to a minimum. An additional 24 working hours for overhauling the hydrogen plant may suffice to make the yearly gain in output due to hydrogen cooling a questionable profit. It should be added that the increase in output cannot be very great, because the temperature rise as compared to that of a generator in which the formation of films of air between the various surfaces is avoided, does not decrease as much as is usually expected and also because in a machine all parts of which are fully utilized, the temperature rise is not the sole factor determining the dimensions. The immediate outlay for the hydrogen plant is at present certainly greater than the saving in material for building the generator.

There is opposition in some quarters from the side of the clients to an increase in output and this under the form of regulations governing short-circuit conditions, i. e. the ratio of short-circuit current at no-load excitation to rated current. In the case of the generator considered here, this ratio is 0·45 and is exactly half that which is required in the U. S. A., for example. As, however, such generators have proved themselves to be absolutely reliable in service, there can be no reason to depart from advantageous conditions for the generator simply to conform to certain customs in judging stability or hypothetical conjectures as to future developments.

By increasing the short-circuit ratio, the suitability of two-pole generators for carrying capacitive load and active load simultaneously is, fundamentally, hardly improved. Under no conditions can an under-excited generator be compared to an over-excited one as regards stability; it would be better to go back to the well-known practice of putting in synchronous condensers, also recommended for big short-circuit ratios, as long as new regulating devices at present being studied are not ready for the market.

With hydrogen cooling, the danger of explosions gave rise to apprehension. To-day, it is rather the somewhat over-elaborated devices, put in to avert this danger, which cause apprehension. At the beginning, these measures were fully justified, as precautionary measures. We think the near future will see simplifications introduced especially as regards shaft glands and some accessory parts.

The hard fight to reach the highest efficiency possible is in realty based on the instinct not to waste natural treasures; in our own special case this can be translated into an urge to save that precious commodity, coal.

(MS 807) *J. Prévost. (Mo.)*

CONTRIBUTION TO THE THEORY OF THE LUBRICATION OF GEARS AND OF THE STRESSING OF THE LUBRICATED FLANKS OF GEAR TEETH.

Decimal index 621.891 : 621.831

The influence of the elastic deformation of the flanks of the gear teeth on the shape of the oil film is investigated theoretically and it is shown that the oil film between elastic cylindrical surfaces is characterized by a single dimensionless number.

The shape of the oil film is calculated for one value of this new characteristic number, and a new formula for the maximum pressure between elastic, lubricated flanks of gear teeth is suggested.

I. INTRODUCTION.

THE results of many investigations dealing with the stressing of tooth flanks and their lubrication have already been published. The stressing is generally calculated according to the theory of Hertz, and the lubrication according to the theory of Reynolds.

Both theories are based on certain assumptions. The theory of Hertz assumes direct contact between the two surfaces pressed together, and therefore no oil film may exist between the two surfaces. The theory of Reynolds, on the other hand, assumes a definite shape for the space between the two surfaces, and therefore, no deformation may occur.

In actual fact neither assumption is true. Modern gears must be well lubricated and the flanks must not come into direct contact. Moreover the stresses are so great that the flanks are considerably deformed. A theory of gear lubrication must, therefore, take into consideration both the oil film and the deformation of the flanks.

This task is difficult but it is possible in various ways to determine the influence of the different factors step by step.

The analysis of the dimensions involved offers the simplest means of finding the essential characteristic numbers which apply to the case being investigated. It is found that two independent characteristic numbers exist. By means of a general calculation of the elastic deformation, it can be shown that the conditions in the actual oil film may be represented by a *single characteristic number,* which is *the product* of the two numbers already mentioned. The further treatment is thus considerably simplified. It is fundamentally possible to calculate pure numbers in function of this single characteristic number which allow the formulae developed according to the two simplified theories to be easily converted and applied to the general case of lubricated, elastic flanks.

II. THE THEORY OF DIMENSIONS.

The following items exercise their influence on the actions which take place between lubricated tooth flanks:—

	Dimension
The mean radius of curvature R . .	m^{+1}
The modulus of elasticity[1] $E' = E/(1-\nu^2)$	$kg^{+1}\ m^{-2}$
The linear loading P/l	$kg^{+1}\ m^{-1}$
The viscosity of the lubricating oil η	$kg^{+1}\ m^{-2}\ s^{+1}$
Average rolling velocity u . . .	$m^{+1}\ s^{-1}$

From these values two dimensionless numbers can be formed, which are independent of one another. Firstly we have

$$\frac{P/l}{E'R} = H \tag{1}$$

This number characterizes the actions which take place between dry, elastic surfaces. Secondly we have

$$\frac{P/l}{\eta \cdot u} = S \tag{2}$$

This number characterizes the actions which take place between rigid, lubricated surfaces. Complete geometrical similarity of two different gears is, therefore, only to be expected if *both H and S* have the same values in both cases.

The nature of the individual laws applying to the lubrication of elastic surfaces cannot be determined from these similarity investigations. The laws are known, on the other hand, for the two special cases mentioned. For dry, elastic surfaces the formulae of Hertz give for the maximum stressing[2]:

$$\sigma_H = \sqrt{\frac{1}{2\pi}}\sqrt{\frac{P \cdot E'}{l \cdot R}} = 0{\cdot}40\,\frac{P/l}{R}\cdot\frac{1}{\sqrt{H}} = 0{\cdot}40\,E'\sqrt{H} \tag{3}$$

and for the width of the surface of contact:

$$b = \sqrt{\frac{8}{\pi}}\sqrt{\frac{P \cdot R}{l \cdot E'}} = 1{\cdot}60 \cdot R\sqrt{H} \tag{4}$$

while for rigid, lubricated surfaces, for example, the following formulae apply[3]: for the minimum thickness of oil film:—

$$h_0 = 4{\cdot}896\,\frac{\eta \cdot u \cdot R}{P/l} = 4{\cdot}896\,R \cdot \frac{1}{S} \tag{5}$$

and for the maximum oil pressure:—

$$p_{max} = 1{\cdot}521\,\frac{\eta \cdot u}{h_0^2}\sqrt{2Rh_0} = 0{\cdot}1985\,\frac{P/l}{R}\sqrt{S} = 0{\cdot}1985\,\frac{\eta \cdot u}{R}\sqrt{S^3} \tag{6}$$

Comparison of the formulae (3) and (6) leads at once to the supposition that similar conditions will exist, if merely the product HS assumes the same value, for we have the relationship

$$\frac{p_{max}}{\sigma_H} = 0{\cdot}50\,\sqrt{H \cdot S} \tag{7}$$

and it may be supposed that, if the ratio of the stressing according to the Hertz formula to the maximum oil pressure according to Reynolds' theory remains the same, the other conditions will also be comparable.

Further investigation shows that this is actually the case.

III. THE ELASTIC DEFORMATION OF THE TOOTH FLANKS.

Since the elastic deformation of the tooth flanks will always be small, it is not necessary in the calculations to take the radius of curvature into consideration; it is permissible to treat the tooth flank as a plane surface.

To determine the deformation y of the plane surface of an infinitely large body at a distance r from a concentrated load P, the formula of Boussinesq[4] may be applied:

$$y = \frac{P}{\pi E'} \cdot \frac{1}{r} \tag{8}$$

If a strip of width $d\xi$ at a distance ξ from and parallel to the Z axis is loaded with a pressure p per unit area, the deformation caused is obtained by integration over the whole strip (Fig. 1) and is given by:—

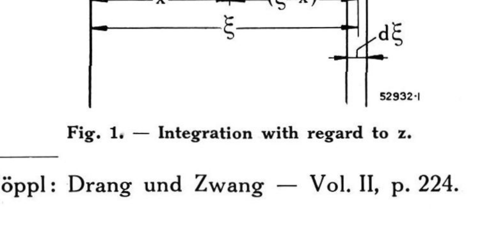


Fig. 1. — Integration with regard to z.

[1] For the case when contraction in one transverse direction is prevented.

[2] Hütte "Des Ingenieurs Taschenbuch", 26th edition, Vol. 1, p. 683.

[3] Engineering — 11th August 1916, p. 120. Heidebroek arrives at quite similar results in "Forschung", July/August 1935, p. 161.

[4] Föppl: Drang und Zwang — Vol. II, p. 224.

$$d[y(x)] = \frac{p \cdot d\xi}{\pi E'} \cdot \int_{-\infty}^{+\infty} \frac{dz}{\sqrt{(\xi - x)^2 + z^2}} \qquad (9)$$

The integral is equal to infinity, but since we are only concerned with the deformation in the vicinity of the load, a usable result may be obtained in the following way. The value of $y'(x)$ is first determined:—

$$d[y'(x)] = \frac{p\, d\xi}{\pi E'} \cdot \int_{-\infty}^{+\infty} \frac{(\xi - x)\, dz}{\left(\sqrt{(\xi - x)^2 + z^2}\right)^3} \qquad (10)$$

The integration can now be carried out and we have

$$\int_{-\infty}^{+\infty} \frac{(\xi - x)\, dz}{\left(\sqrt{(\xi - x)^2 + z^2}\right)^3} = \frac{2}{\xi - x} \qquad (11)$$

and consequently

$$d[y'(x)] = \frac{2p \cdot d\xi}{\pi E'} \cdot \frac{1}{\xi - x} \qquad (12)$$

It then follows that [1]

$$d[y(x)] = -\frac{2p \cdot d\xi}{\pi E'} \cdot \log_e(\xi - x) \qquad (13)$$

If the load is distributed over a strip of finite width, in which case $p(\xi)$ may be a function of ξ, the total deformation may be found by a further integration with regard to ξ (Fig. 2).

$$y(x) = -\frac{2}{\pi E'} \int_{-\infty}^{+\infty} p(\xi) \cdot \log(\xi - x)\, d\xi \qquad (14)$$

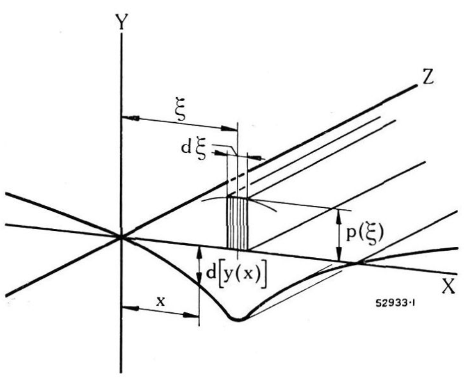


Fig. 2. — Integration with regard to ξ.

[1] From a physical viewpoint it would, indeed, be more correct to write $\int \frac{2\,dx}{x} = \log_e(x^2)$ because the elastic deformation actually occurs for both positive and negative values of x and must, therefore, be real. The method of expression chosen is, however, more convenient for the calculations and yields the same results if at all times only the absolute value of the number is taken.

With this formula the deformation of a tooth flank caused by any distributed load can be determined.

IV. CALCULATIONS FOR AN OIL FILM BETWEEN ELASTIC SURFACES.

It is now possible to calculate the shape of an oil film between elastic surfaces. In order that the calculation may be quite general, we shall introduce dimensionless coordinates. For the unit of length $\sqrt{2Rh_0}$ is chosen, and for the unit of film thickness[2] h_0.

We may then define the following dimensionless coordinates:—

$$\begin{aligned} v &= \xi/\sqrt{2Rh_0} \\ w &= x/\sqrt{2Rh_0} \\ e &= h/h_0 \end{aligned} \qquad (15)$$

The following dimensionless expression for the pressure is also introduced[3]:—

$$\psi(v) = \int_{-\infty}^{v} \frac{e(v) - a}{[e(v)]^3}\, dv \qquad (16)$$

where a is a constant of integration. From the equation for the pressure in the oilfilm [3]

$$\frac{dp}{d\xi} = -12\eta u \frac{h - c}{h^3}$$

it follows directly that

$$p(\xi) = -\frac{12\eta u}{h_0^2} \sqrt{2Rh_0} \cdot \psi(v) \qquad (17)$$

Fig. 3 shows the variation of ψ for an oil film between rigid, cylindrical surfaces.

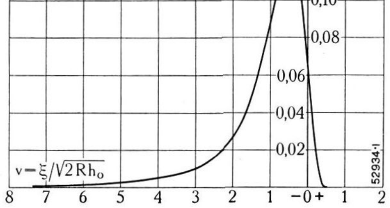


Fig. 3. — Pressure distribution in an oil film between rigid, cylindrical rollers. (From "Engineering".)

[2] That two units of length must be chosen is due to the fact that complete geometrical similarity must be abandoned. See Chapter V, p. 377.

[3] Engineering l.c. with somewhat different designations.

We may now substitute for $p(\xi)$ in equation (14). Since *both* boundary surfaces of the oil film are deformed by the pressure, the alteration of the film thickness $\Delta h = 2y$ and we have[1]

$$\frac{2y}{h_0} = \Delta e(w) \tag{18}$$
$$= \frac{4}{\pi E'} \cdot \frac{12\eta u}{h_0^3} \left(\sqrt{2Rh_0}\right)^2 \int_{-\infty}^{+\infty} \psi(v) \log_e (v - w)\, dv$$

On the other hand the linear pressure[2] is given by

$$P/l = \int p(\xi)\, d\xi = \frac{12\eta u}{h_0^2} \left(\sqrt{2Rh_0}\right)^2 \int_{-\infty}^{+\infty} \psi(v)\, dv \tag{19}$$

and after elimination of h_0 between (18) and (19)

$$\Delta e(w) = \frac{1}{6\pi} \cdot \frac{(P/l)^2}{E' \cdot R \cdot \eta \cdot u} \cdot \frac{\int_{-\infty}^{+\infty} \psi(v) \log_e (v-w)\, dv}{\left[\int_{-\infty}^{+\infty} \psi(v)\, dv\right]^2}$$

or

$$\Delta e(w) = \frac{1}{6\pi} \cdot H \cdot S \cdot \frac{\int_{-\infty}^{+\infty} \psi(v) \log_e (v-w)\, dv}{\left[\int_{-\infty}^{+\infty} \psi(v)\, dv\right]^2} \tag{20}$$

In making the calculation, the procedure is then as follows. A shape of the gap $e(v)$ is *assumed* and $\psi(v)$ is determined from equation (16) by integration, with the usual limit conditions $\psi(-\infty) = 0$ for the beginning of the pressure zone, and besides $\psi = 0$ also $\psi' = 0$ for the end of the pressure zone, i. e. the curve of ψ touches the v axis (see Fig. 3). The value of the integration constant a in equation (16) is determined by this latter condition.

The integrals

$$A = \int_{-\infty}^{+\infty} \psi(v)\, dv \quad \text{and} \quad B(w) = \int_{-\infty}^{+\infty} \psi(v) \log_e (v-w)\, dv \tag{21}$$

are now calculated, and subsequently the elastic deformation

$$\Delta e(w) = \frac{1}{6\pi} \cdot HS \cdot \frac{B(w)}{A^2} \tag{22}$$

is determined. The original shape of the gap between two cylinders is approximately (if now instead of v the variable w is inserted)

$$e_0 = 1 + w^2 \tag{23}$$

and the width of the deformed gap is, therefore

$$e(w) = 1 + w^2 - \Delta e(w) \tag{24}$$

If the newly calculated gap width is the same at all points as originally assumed, the correct solution has been found, otherwise the process must be repeated.

V. CONCLUSIONS.

It may be seen from equation (20) that the shape of an oil film bounded by elastic surfaces is determined exclusively by the value of the product HS. The supposition mentioned in connection with equation (7) is, therefore, proved. *If two gears are to be comparable, the product of the two characteristic numbers H and S*

$$HS = \frac{(P/l)^2}{E' R \eta u} \tag{25}$$

must have the same value; the number HS thus characterizes the behaviour of an elastic, lubricated tooth flank.

When the product HS is here introduced as a new characteristic number, this does not contradict the conclusion drawn from the theory of similarity, mentioned at the beginning, according to which *both H and S individually* must have the same values for *complete* similarity. In developing the above equations complete geometrical similarity was not demanded, in that the ratio h_0/R does not remain constant. This is, however, permissible since the conditions in the immediate vicinity of the pressure zone are determined by the ratio $x/\sqrt{2Rh_0}$ and not by the ratio h_0/R.

Both the pressure curve $\psi(v)$ and the shape of the oil film $e(w)$ are completely defined by the value of HS. It follows from this that all the formulae (3) to (6) will also be valid for elastic, lubricated surfaces, the only difference being that the constants are replaced by functions of HS.

More exact information can be obtained by calculating the shape of the oil film for various values of HS. If such a calculation has been carried out, i. e. if the curve $\psi(v)$ is known for a given value of HS, the ratio p_{max}/σ_H may be calculated in the following manner:

From equation (17) the maximum oil pressure is

$$p_{max} = \frac{12\eta u}{h_0^2} \sqrt{2Rh_0} \cdot \psi_{max}$$

and from equations (19) and (21)

$$P/l = \frac{12\eta u}{h_0^2} \left(\sqrt{2Rh_0}\right)^2 \cdot A \text{ where } A = \int_{-\infty}^{+\infty} \psi(v)\, dv$$

From these two equations h_0 can be eliminated:

$$p_{max} = \sqrt{\frac{(P/l)^3}{48 R^2 \eta u}} \cdot \frac{\psi_{max}}{\sqrt{A^3}}$$

[1] The change of sign relative to (14) is due to the fact that $\psi(v)$ has been made positive for pressures, in contrast to $p(\xi)$.

[2] Engineering l. c.

and with the help of equation (3) we obtain the equation

$$\frac{p_{max}}{\sigma_H} = \sqrt{\frac{\pi}{24}} \cdot \sqrt{\frac{(P/l)^2}{E' R \eta u}} \cdot \frac{\psi_{max}}{\sqrt{A^3}} = \sqrt{\frac{\pi}{24}} \cdot \sqrt{H \cdot S} \cdot \frac{\psi_{max}}{\sqrt{A^3}} \qquad (26)$$

For the time being the deformation and the pressure distribution (Fig. 4) have been calculated for a value

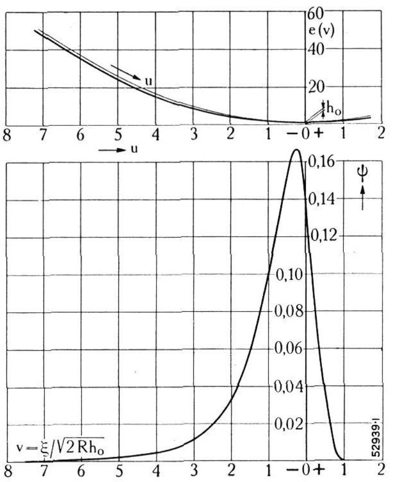


Fig. 4. — Pressure distribution in an oil film between elastic rollers for HS ≅ 2.
——— Original shape of the oil film.
——— Deformed shape of the oil film.

HS ≅ 2, and from this the corresponding value of p_{max}/σ_H. The calculation requires considerable patience and the convergence is not particularly good even for this small value of HS. To proceed further with the calculation in this form would only have an insignificant value because, in practice, the viscosity of the lubricating oil increases to such an extent with increasing pressure, that the pressure distribution in the oil film would be considerably affected.

To set up a formula for p_{max}/σ_H on the basis of a single calculated point is only possible with certain reservations. Since, however, in addition to the calculated point, the limit conditions are known, the attempt will nevertheless be made.

The limit conditions are the following. For large values of HS, p_{max}/σ_H must obviously be equal to unity. For small values of HS, on the other hand, equation (7), $p_{max}/\sigma_H = 0{\cdot}50 \sqrt{HS}$ must apply. In addition the calculated value of p_{max}/σ_H for HS ≅ 2 must lie on the curve.

A simple formula which satisfies these conditions well is

$$\frac{p_{max}}{\sigma_H} = \sqrt{\frac{0{\cdot}25\,HS}{0{\cdot}25\,HS + 1}} \qquad (27)$$

where σ_H is to be calculated according to equation (3). As may be seen, this equation fulfils the limit conditions for large and small values of HS, and is in comparatively close agreement with the calculated point. Fig. 5 shows that equation (27) will give values which are somewhat too high.

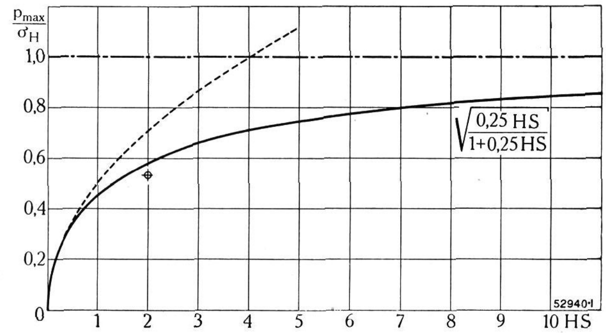


Fig. 5. — Relationship between maximum oil pressure in the oil film and the maximum compressive stress according to the formula of Hertz.
The curve represents the formula suggested

$$\frac{p_{max}}{\sigma_H} = \sqrt{\frac{0{\cdot}25\,HS}{1+0{\cdot}25\,HS}}$$

The point shown was calculated from equations (16) and (20).

There is still one point which deserves mention. If E, u and η may be regarded as constant, e. g. on account of the design practice of a firm, we may derive directly from the condition HS = constant the relationship

$$P/l \text{ is proportional to } \sqrt{R}$$

i. e. *Parsons' formula.* So long as $\eta \cdot u$ remains constant, the well-known formula of Parsons gives equivalent conditions in the oil film.

VI. THE NUMERICAL CALCULATION.

One difficulty is encountered in carrying out the numerical calculation, namely that the expression to be integrated in equation (18) is equal to infinity when v = w. The integral itself is indeed finite but the integration is consequently inconvenient and may be replaced with advantage by a sum. For this purpose the pressure diagram is divided into strips of finite width b, and the total deformation is obtained as the sum of all the deformations produced by the separate strips.

The integration over the individual strips can be carried out mathematically so that the points where the expression to be integrated becomes infinitely large may be completely avoided.

Within such a strip the function can be replaced by a series. Let the middle of the strip have the abscissa v_0. Then

$$\psi(v) = \psi(v_0) + \psi'(v_0)\frac{v-v_0}{1!} + \psi''(v_0)\frac{(v-v_0)^2}{2!} + \ldots \qquad (28)$$

and the integral (18) can now be calculated term for term.

By means of the following trick the convergency can be considerably improved. The even terms $\psi''(v_0) \cdot (v-v_0)^2/2!$ etc. all give a resultant force, because

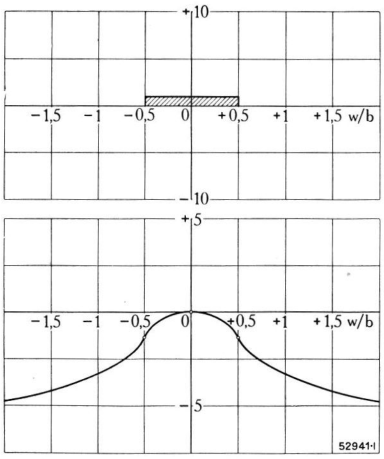


Fig. 6. — **Deformation of the plane surface of an elastic body for strip loading p = 1**

$$f_0\left(\frac{w}{b}\right) = -\left\{\frac{2w}{b}\log_e\frac{2w/b+1}{2w/b-1} + \log_e\left(4w^2/b^2-1\right)\right\}$$

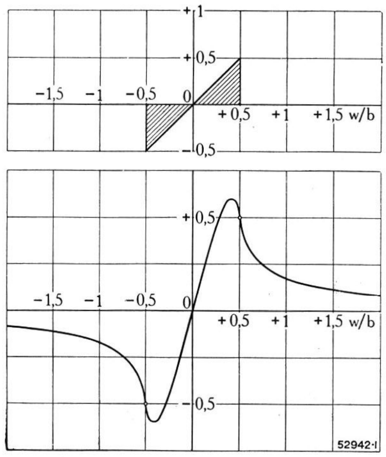


Fig. 7. — **Deformation of the plane surface of an elastic body for strip loading p = w/b**

$$f_1\left(\frac{w}{b}\right) = -\frac{1}{4}\left\{\left[\left(\frac{2w}{b}\right)^2-1\right]\times\log_e\frac{2w/b+1}{2w/b-1} - 2\left(\frac{2w}{b}\right)\right\}$$

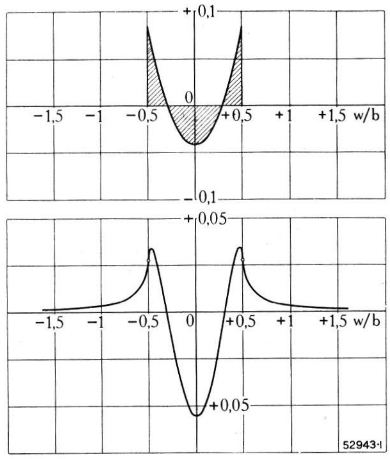


Fig. 8. — **Deformation of the plane surface of an elastic body for strip loading $p = 1/2\,(w/b)^2 - 1/24$**

$$f_2\left(\frac{w}{b}\right) = -\frac{1}{24}\left\{\frac{2w}{b}\left[\left(\frac{2w}{b}\right)^2-1\right]\times\log_e\frac{2w/b+1}{2w/b-1} - 2\left(\frac{2w}{b}\right)^2 + \frac{4}{3}\right\}$$

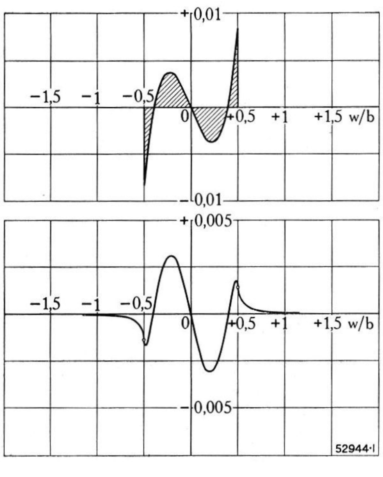


Fig. 9. — **Deformation of the plane surface of an elastic body for strip loading $p = 1/6\,(w/b)^3 - 1/40\,(w/b)$**

$$f_3\left(\frac{w}{b}\right) = -\frac{1}{192}\left\{\left[\left(\frac{2w}{b}\right)^4-1\right]\log_e\frac{2w/b+1}{2w/b-1} - 2\left(\frac{2w}{b}\right)^3 - \frac{2}{3}\left(\frac{2w}{b}\right) - \frac{24}{5}f_1\left(\frac{w}{b}\right)\right\}$$

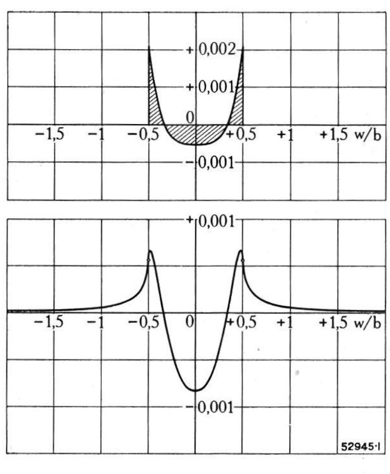


Fig. 10. — **Deformation of the plane surface of an elastic body for strip loading $p = 1/24\,(w/b)^4 - 1/1920$**

$$f_4\left(\frac{w}{b}\right) = -\frac{1}{1920}\left\{\frac{2w}{b}\left[\left(\frac{2w}{b}\right)^4-1\right]\log_e\frac{2w/b+1}{2w/b-1} - 2\left(\frac{2w}{b}\right)^4 - \frac{2}{3}\left(\frac{2w}{b}\right)^2 + \frac{8}{5}\right\}$$

$$\int_{v_0-\frac{b}{2}}^{v_0+\frac{b}{2}} \psi^{(n)}(v_0)\,\frac{(v-v_0)^n}{n!}\,dv = \frac{b}{(n+1)!}\,\psi^{(n)}(v_0)\left(\frac{b}{2}\right)^n \quad (29)$$

for even values of n. It is, therefore, advantageous to group this force with the first term $\psi(v_0)$. The odd terms $\psi'''(v_0)\cdot(v-v_0)^3/3!$ etc. behave in a similar manner. They all give a couple because

$$\int_{v_0-\frac{b}{2}}^{v_0+\frac{b}{2}} \psi^{(n)}(v_0)\,\frac{(v-v_0)^n}{n!}\,(v-v_0)\,dv = \frac{b^3}{12}\cdot\frac{3}{n!\,(n+2)}\,\psi^{n}(v_0)\left(\frac{b}{2}\right)^{n-1} \quad (30)$$

for odd values of n. These couples may be advantageously grouped with the second term $\psi'(v_0)$. Equation (28) now becomes

$$\psi(v)=\psi(v_0)+\frac{1}{3!}\psi''(v_0)\left(\frac{b}{2}\right)^2+\frac{1}{5!}\psi^{(4)}(v_0)\left(\frac{b}{2}\right)^4+\dots$$
$$+\left[\psi'(v_0)+\frac{3}{3!\cdot5}\psi'''(v_0)\left(\frac{b}{2}\right)^2+\frac{3}{5!\cdot7}\psi^{(5)}(v_0)\left(\frac{b}{2}\right)^4+\dots\right]\frac{v-v_0}{1!}$$
$$+\psi''(v_0)\left[\frac{(v-v_0)^2}{2!}-\frac{1}{3!}\left(\frac{b}{2}\right)^2\right] \tag{31}$$
$$+\psi'''(v_0)\left[\frac{(v-v_0)^3}{3!}-\frac{3}{3!\cdot5}\left(\frac{b}{2}\right)^2(v-v_0)\right]$$
$$+\psi^{(4)}(v_0)\left[\frac{(v-v_0)^4}{4!}-\frac{1}{5!}\left(\frac{b}{2}\right)^4\right]$$
$$+\psi^{(5)}(v_0)\left[\frac{(v-v_0)^5}{5!}-\frac{3}{5!\cdot7}\left(\frac{b}{2}\right)^4(v-v_0)\right]+\dots$$

In this manner the total resultant force is represented by the first term and the total torque by the second term. The following terms, referred to the total width of strip, give neither resultant force nor torque, so that the remote effect of these terms is exceedingly small. They only provide a small contribution to the deformation in the immediate neighbourhood of the strip.

The integral (18) can now be evaluated term by term. It is sufficient to calculate the separate integrals for $v_0 = 0$, since, for other values of v_0, the curve calculated in this manner can be simply displaced parallel to itself in such a manner that it always passes through the zero point.

The first integral is then

$$\int_{-\frac{b}{2}}^{+\frac{b}{2}}\log_e(v-w)\,dv=\frac{b}{2}\left\{\frac{2w}{b}\log_e\frac{2w/b+1}{2w/b-1}+\log_e(4w^2/b^2-1)\right\}=-\frac{1}{2}b\,f_0\left(\frac{w}{b}\right) \tag{32}$$

The constant of integration is so chosen that $f_0(0)=0$, because $f_0(\infty)\rightarrow-\infty$. The new limits are due to the fact that the loading disappears outside the strip from $-b/2$ to $+b/2$.

The second integral is

$$\int_{-\frac{b}{2}}^{+\frac{b}{2}}v\log_e(v-w)\,dv=\frac{1}{2}\left(\frac{b}{2}\right)^2\left\{\left[\left(\frac{2w}{b}\right)^2-1\right]\log_e\frac{2w/b+1}{2w/b-1}-2\left(\frac{2w}{b}\right)\right\}=-\frac{1}{2}b^2 f_1\left(\frac{w}{b}\right) \tag{33}$$

$f_1\left(\frac{w}{b}\right)$ disappears both for $\frac{w}{b}=0$ and $\frac{w}{b}=\infty$.

The general forms of the integrals of the higher terms are as follows:—

1. For n even and greater than 0:

$$\int_{-\frac{b}{2}}^{+\frac{b}{2}}\left[\frac{v^n}{n!}-\frac{(b/2)^n}{(n+1)!}\right]\log_e(v-w)\,dv$$
$$=\frac{(b/2)^{n+1}}{(n+1)!}\left\{\frac{2w}{b}\left[\left(\frac{2w}{b}\right)^n-1\right]\log_e\frac{2w/b+1}{2w/b-1}\right.$$
$$-2\left[\left(\frac{2w}{b}\right)^n+\frac{1}{3}\left(\frac{2w}{b}\right)^{n-2}+\frac{1}{5}\left(\frac{2w}{b}\right)^{n-4}+\dots\right.$$
$$\left.\left.+\frac{1}{n-1}\left(\frac{2w}{b}\right)^2-\frac{n}{n+1}\right]\right\}=-\frac{1}{2}b^{n+1}f_n\left(\frac{w}{b}\right) \tag{34}$$

2. For n odd and greater than 1:

$$\int_{-\frac{b}{2}}^{+\frac{b}{2}}\left[\frac{v^n}{n!}-\frac{3}{n!\,(n+2)}v\right]\log_e(v-w)\,dv$$
$$=\frac{(b/2)^{n+1}}{(n+1)!}\left\{\left[\left(\frac{2w}{b}\right)^{n+1}-1\right]\log_e\frac{2w/b+1}{2w/b-1}\right.$$
$$-2\left[\left(\frac{2w}{b}\right)^n+\frac{1}{3}\left(\frac{2w}{b}\right)^{n-2}+\frac{1}{5}\left(\frac{2w}{b}\right)^{n-4}+\dots\right.$$
$$\left.\left.+\frac{1}{n}\left(\frac{2w}{b}\right)\right]+6\frac{n+1}{n+2}f_1\left(\frac{w}{b}\right)\right\}=-\frac{1}{2}b^{n+1}f_n\left(\frac{w}{b}\right) \tag{35}$$

The problem is thus fundamentally solved. The following expansions are convenient for the numerical calculations:

1. $n = 0$

$$f_0\left(\frac{w}{b}\right)=-2\log_e\left(\frac{2w}{b}\right)-2+\left[\frac{1}{1\cdot3}\left(\frac{b}{2w}\right)^2+\frac{1}{2\cdot5}\left(\frac{b}{2w}\right)^4+\frac{1}{3\cdot7}\left(\frac{b}{2w}\right)^6+\dots\right] \tag{36}$$
$$=-2\log_e\left(\frac{2w}{b}\right)-2+\sum_{k=1}^{\infty}\frac{1}{k(2k+1)}\left(\frac{b}{2w}\right)^{2k}$$

2. $n = 1$

$$f_1\left(\frac{w}{b}\right)=\left[\frac{1}{1\cdot3}\left(\frac{b}{2w}\right)+\frac{1}{3\cdot5}\left(\frac{b}{2w}\right)^3+\frac{1}{5\cdot7}\left(\frac{b}{2w}\right)^5+\dots\right] \tag{37}$$
$$=\sum_{k=1}^{\infty}\frac{1}{(2k-1)(2k+1)}\left(\frac{b}{2w}\right)^{2k+1}$$

3. n even and greater than 0:

$$f_n\left(\frac{w}{b}\right)=\frac{2n}{(n+1)!}\frac{1}{2^n}\left[\frac{1}{3(n+3)}\left(\frac{b}{2w}\right)^2+\frac{1}{5(n+5)}\left(\frac{b}{2w}\right)^4+\frac{1}{7(n+7)}\left(\frac{b}{2w}\right)^6+\dots\right] \tag{38}$$
$$=\frac{2n}{(n+1)!}\frac{1}{2^n}\sum_{k=1}^{\infty}\frac{1}{(2k+1)(n+2k+1)}\left(\frac{b}{2w}\right)^{2k}$$

TABLE I

$f_0\left(\frac{w}{b}\right) \qquad f_0\left(-\frac{w}{b}\right)=f_0\left(\frac{w}{b}\right)$

$\frac{w}{b}$	f_0	$\frac{w}{b}$	f_0
0	0	10	−7·990631
0·5	−1·386294	10·5	−8·088289
1	−3·295837	11	−8·181396
1·5	−4·158883	11·5	−8·270358
2	−4·751353	12	−8·355529
2·5	−5·205379	12·5	−8·437218
3	−5·574182	13	−8·515700
3·5	−5·884976	13·5	−8·591216
4	−6·153650	14	−8·663984
4·5	−6·390319	14·5	−8·734195
5	−6·601827	15	−8·802024
5·5	−6·793029	15·5	−8·867627
6	−6·967494	16	−8·931146
6·5	−7·127923	16·5	−8·992709
7	−7·276411	17	−9·052433
7·5	−7·414617	17·5	−9·110426
8	−7·543874	18	−9·166781
8·5	−7·665272	18·5	−9·221592
9	−7·779714	19	−9·274941
9·5	−7·887954	19·5	−9·326904
10	−7·990631	20	−9·377551

TABLE II

$f_1\left(\frac{w}{b}\right) \qquad f_1\left(-\frac{w}{b}\right)=-f_1\left(\frac{w}{b}\right)$

$\frac{w}{b}$	f_1	$\frac{w}{b}$	f_1
0	0	10	0·016675
0·5	0·500000	10·5	0·015880
1	0·176041	11	0·015158
1·5	0·113706	11·5	0·014498
2	0·084404	12	0·013894
2·5	0·067209	12·5	0·013338
3	0·055868	13	0·012824
3·5	0·047815	13·5	0·012349
4	0·041798	14	0·011908
4·5	0·037129	14·5	0·011497
5	0·033400	15	0·011114
5·5	0·030353	15·5	0·010755
6	0·027816	16	0·010419
6·5	0·025671	16·5	0·010103
7	0·023834	17	0·009806
7·5	0·022242	17·5	0·009525
8	0·020850	18	0·009261
8·5	0·019621	18·5	0·009010
9	0·018530	19	0·008773
9·5	0·017554	19·5	0·008548
10	0·016675	20	0·008334

TABLE III

$f_2\left(\frac{w}{b}\right) \qquad f_2\left(-\frac{w}{b}\right)=f_2\left(\frac{w}{b}\right)$

$\frac{w}{b}$	f_2	$\frac{w}{b}$	f_2
0	−0·055556	10	0·000028
0·5	+0·027778	10·5	0·000025
1	0·003125	11	0·000023
1·5	0·001297	11·5	0·000021
2	0·000714	12	0·000019
2·5	0·000452	12·5	0·000018
3	0·000312	13	0·000016
3·5	0·000229	13·5	0·000015
4	0·000175	14	0·000014
4·5	0·000138	14·5	0·000013
5	0·000112	15	0·000012
5·5	0·000092	15·5	0·000012
6	0·000077	16	0·000011
6·5	0·000066	16·5	0·000010
7	0·000057	17	0·000010
7·5	0·000049	17·5	0·000009
8	0·000043	18	0·000009
8·5	0·000039	18·5	0·000008
9	0·000034	19	0·000008
9·5	0·000031	19·5	0·000007
10	0·000028	20	0·000007

TABLE IV

$f_3\left(\frac{w}{b}\right) \qquad f_3\left(-\frac{w}{b}\right)=-f_3\left(\frac{w}{b}\right)$

$\frac{w}{b}$	f_3	$\frac{w}{b}$	f_3
0	0	3	0·000002
0·5	0·001389	3·5	0·000001
1	0·000048	4	0·000001
1·5	0·000013	4·5	0·000000
2	0·000005	5	
2·5	0·000003		
3	0·000002		

TABLE V

$f_4\left(\frac{w}{b}\right) \qquad f_4\left(-\frac{w}{b}\right)=f_4\left(\frac{w}{b}\right)$

$\frac{w}{b}$	f_4	$\frac{w}{b}$	f_4
0	−0·000833	5	0·000002
0·5	+0·000556	5·5	0·000002
1	0·000056	6	0·000001
1·5	0·000023	6·5	0·000001
2	0·000013	7	0·000001
2·5	0·000008	7·5	0·000001
3	0·000006	8	0·000001
3·5	0·000004	8·5	0·000001
4	0·000003	9	0·000001
4·5	0·000002	9·5	0·000001
5	0·000002	10	0·000000

4. n odd and greater than 1:

$$f_n\left(\frac{w}{b}\right)=\frac{4\,(n^2-1)}{(n+2)!}\,\frac{1}{2^n}\left[\frac{1}{3\cdot5\cdot(n+4)}\left(\frac{b}{2\,w}\right)^3+\frac{2}{5\cdot7\cdot(n+6)}\left(\frac{b}{2\,w}\right)^5+\ldots\right] \tag{39}$$

$$=\frac{4\,(n^2-1)}{(n+2)!}\,\frac{1}{2^n}\sum_{k=1}^{\infty}\frac{k}{(2k+1)\,(2k+3)\,(n+2k+2)}\left(\frac{b}{2\,w}\right)^{2k+1}$$

With these functions the integral (18) can now be written in the following form as a sum, in which $v=kb$ where k is a whole number.

$$\begin{aligned} B(w)=b\sum_{k=-\infty}^{+\infty}\Big\{\bar{\psi}\;(kb)&\left[f_0\left(\frac{w}{b}-k\right)-f_0(-k)\right]\\ &+b\;\bar{\psi}'\;(kb)\left[f_1\left(\frac{w}{b}-k\right)-f_1(-k)\right]\\ &+b^2\,\psi''\,(kb)\left[f_2\left(\frac{w}{b}-k\right)-f_2(-k)\right]\\ &+\ldots\Big\}\\ =b\sum_{n=o}^{\infty}b^n&\sum_{k=-\infty}^{k=+\infty}\psi^{(n)}\,(kb)\;F_n\left(\frac{w}{b};\,k\right) \end{aligned} \tag{40}$$

where

$$F_n\left(\frac{w}{b};\, k\right) \equiv f_n\left(\frac{w}{b} - k\right) - f_n(-k)$$

and the designations $\overline{\psi}$ (kb) and $\overline{\psi}'$ (kb) are introduced as abbreviations for the two first terms of equation (31). Since the loading will be generally given in the form of tables, it is of advantage to calculate with difference series, and best of all with Stirling's series [1].

$$\begin{aligned}\overline{\psi}(kb) = \psi(kb) &+ 0{\cdot}0417\, \Delta^2 \psi\big((k-1)\,b\big) \\ &- 0{\cdot}0030\, \Delta^4 \psi\big((k-2)\,b\big) \\ &+ 0{\cdot}0039\, \Delta^6 \psi\big((k-3)\,b\big) \\ &- \ldots\end{aligned}$$

$$\begin{aligned}b\,\overline{\psi}'(kb) = &\ \frac{\Delta\,\psi(kb) + \Delta\,\psi\big((k-1)\,b\big)}{2} \\ &- 0{\cdot}1417\, \frac{\Delta^3 \psi\big((k-1)\,b\big) + \Delta^3 \psi\big((k-2)\,b\big)}{2} \\ &+ 0{\cdot}0273\, \frac{\Delta^5 \psi\big((k-2)\,b\big) + \Delta^5 \psi\big((k-3)\,b\big)}{2} \\ &- \ldots\end{aligned}$$

$$\begin{aligned}b^2\,\psi''(kb) = &\ \Delta^2 \psi\big((k-1)\,b\big) \\ &- 0{\cdot}0833\Delta^4 \psi\big((k-2)\,b\big) + \ldots\end{aligned} \qquad (41)$$

$$\begin{aligned}b^3\,\psi'''(kb) = &\ \frac{\Delta^3 \psi\big((k-1)\,b\big) + \Delta^3 \psi\big((k-2)\,b\big)}{2} \\ &- 0{\cdot}25\, \frac{\Delta^5 \psi\big((k-2)\,b\big) + \Delta^5 \psi\big((k-3)\,b\big)}{2} \\ &+ \ldots\end{aligned}$$

$$\begin{aligned}b^4\,\psi^{(4)}(kb) = &\ \Delta^4 \psi\big((k-2)\,b\big) \\ &- 0{\cdot}1667\, \Delta^6 \psi\big((k-3)\,b\big) + \ldots\end{aligned}$$

etc.

The calculation in accordance with equation (40) can be summarized in the form of a scheme with general validity in which the numerical values of the expressions $f_n\,(w/b - k) - f_n\,(-k) = F_n\,(w/b\,;\,k)$ are tabulated once for all. F_0 represents the main factor, F_1 contributes a certain amount, while the higher terms only give small corrections in the immediate vicinity of the load.

(MS 808) *A. Meldahl. (D. S.)*

[1] See "Hütte" 26th edition, Vol. I, p. 178.

ON STEAM-JET AIR EJECTORS.

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It is shown that as complete condensation as possible of the steam jet in the first stage is desirable in multi-stage steam-jet air ejectors. For this reason big coolers are worth the expense of putting in, in the majority of cases.

If water-jet air ejectors are still in use in steam power stations to exhaust air and to maintain vacuum, the reason is to be sought in the special advantage inherent to this type of apparatus, namely, that the water of the jet condenses the last traces of steam remaining in the steam-air mixture drawn off, which allows of saving on compression work. Notably, at partial load when, as a rule, relatively big quantities of air have to be exhausted and when compression work to atmosphere has to be done from a higher state of vacuum, the said advantage can become the decisive factor, in the choice of the type of ejector used. From a purely economic point of view, the steam-jet ejector and the water-jet ejector are about equal, to-day. The steam-jet air ejector, being a more recent technical innovation, is however liable to be considerably developed and it presents the attraction that its working medium, steam, is applied without transformation while, with the water-jet air ejector, the requisite power has first to be produced by turbine, generator, motor and pump.

We have already reached a stage, with the steam-jet air ejector, at which the apparatus can be calculated and built on the basis of reliable figures gained by experience and with the help of generally recognized formulae. By proper dimensioning of the nozzles the earlier difficulties of stabilization can be considered as being definitely eliminated. There is one important part of the steam-jet air ejector, however, on the subject of which some confusion still remains, namely as to what the most advantageous size of cooler is.

It should first be said that the most efficaceous and the most direct method of cooling is not by means of surface coolers but by mixing with the cooling water. Despite this, mixed coolers made no headway because of the complications inherent to leading the water back to the condenser again. For this reason, the surface cooler was given preference as being the easiest to operate and the most reliable. Nevertheless, nothing definite is generally known as to what size it should be in order to work under the best economical conditions. For this reason we give here, by means of an example, the necessary data to allow of estimating the cooler.

Fig. 1 shows various conditions in a surface cooler, inserted between the first and the second steam-jet air ejector, in function of the cooling surface which the mixture of air and steam passes over. The abscissae are the value O/D, that is the size of the cooling surface in m^2 for 1 kg of jet-steam per hour. Here it has been assumed that the cooling-water temperature is 26 ^{0} C, the total gas pressure in this stage 0·128 kg/cm^2 abs and the mixture ratio of air to jet-steam to 1 kg of jet steam is shown by curve 3. Due to the extraction of heat in the cooler, the temperature of the mixture drops, according to curve 4 and almost reaches the temperature of the cooling water 5, while, as condensation increases, the ratio of the weight of condensed steam to that of steam injected 6 approaches unity.

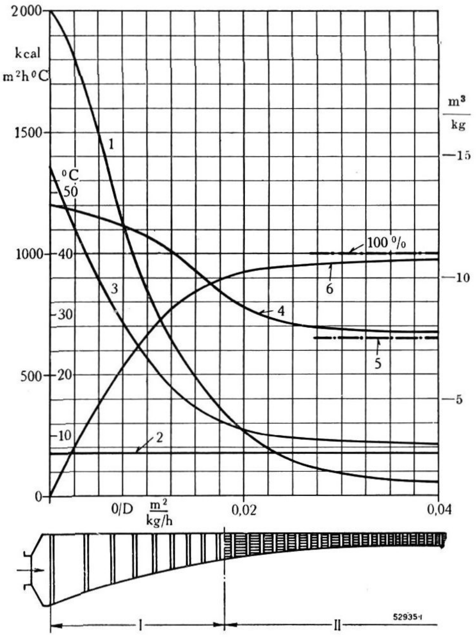

Fig. 1. — Conditions in an intermediate cooler of a two-stage steam-jet air ejector in function of the cooling surface used for 1 kg of jet steam.

Curve 1: Heat-transfer coefficient in kcal/m² h° C.
Curve 2: Volumes of exhausted air referred to 1 kg of jet steam.
Curve 3: Volumes of the mixture of air, jet steam and some traces of steam in m³ referred to 1 kg of jet steam.
Curve 4: Temperature of mixture with a cooling water temperature according to curve 5.
Curve 6: Ratio of weight of steam condensed to weight of jet steam.

Curve 3 shows the drop in the quantity of mixture to be compressed in the next stage by new injection of steam. The bigger the intermediate cooler the lower the work of compression of the second ejector stage and the smaller, also, the amount of jet steam it will need. The coolers have smooth tubes in the first part I and ribbed tubes in the second part II.

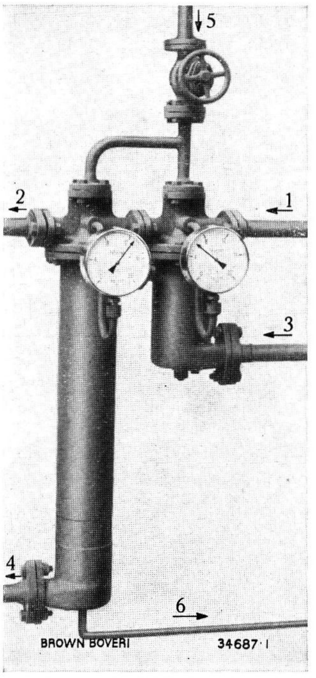

Fig. 2. — Two-stage steam-jet air ejector with small intermediate cooler.

1. Suction connecting branch.
2. Air outlet.
3. Cooling-water inlet.
4. Cooling-water outlet.
5. Jet steam.
6. Condensate of both stages.

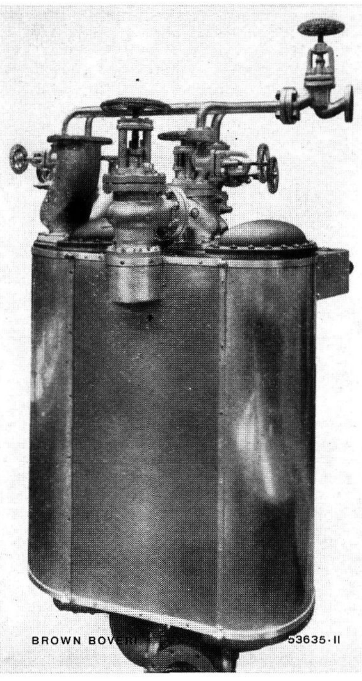

Fig. 3. — Two-stage steam-jet air ejector with big intermediate cooler.

Figs. 2 and 3 differ essentially in the size of the intermediate coolers and, therefore, in their steam consumption. The bigger intermediate cooler results in a more economical but somewhat more expensive steam-jet air ejector.

(weight ratio) at the inlet 0·25. Further, it is assumed that the velocity of the mixture between the cooling surfaces is constant and amounts to 10 m/s which gives a falling heat transfer coefficient, according to curve 1, corresponding to increasing water content in the mixture and the decrease in the partial pressure of the steam.

The quantity of air to be exhausted referred to 1 kg of jet steam is constant and shown by curve 2, the volume of the mixture of air and jet steam referred

Curve 3 gives the size of the cooler. The bigger the cooling surface chosen the more complete is the condensation of the jet steam of the first steam-jet apparatus and the smaller the volume of the mixture to be compressed and the quantity of steam which will be required in the second steam ejector apparatus to deliver the air to atmosphere. It is now a question where the greatest economy lies:— in the purchase of a cheap ejector apparatus with a small cooler or in reducing the expenditure of steam to the jet. In the majority of cases the more expensive apparatus leads to the most economical operation. The comparison of an old steam jet air ejector (Fig. 2) and a new design of the apparatus (Fig. 3) shows the considerable increase in the size of the cooler.

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