A consistent approach to estimate the breakdown voltage of high voltage electrodes under positive switching impulses

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The main propose of this paper is to present a physical model of long air gap electrical discharges under positive switching impulses. The development and progression of discharges in long air gaps are attributable to two intertwined physical phenomena, namely, the leader channel and the streamer zone. Experimental studies have been used to develop empirical and physical models capable to represent the streamer zone and the leader channel. The empirical ones have led to improvements in the electrical design of high voltage apparatus and insulation distances, but they cannot take into account factors associated with fundamental physics and/or the behavior of materials. The physical models have been used to describe and understand the discharge phenomena of laboratory and lightning discharges. However, because of the complex simulations necessary to reproduce real cases, they are not in widespread use in the engineering of practical applications. Hence, the aim of the work presented here is to develop a model based on physics of the discharge capable to validate and complement the existing engineering models. The model presented here proposes a new geometrical approximation for the representation of the streamer and the calculation of the accumulated electrical charge. The model considers a variable streamer region that changes with the temporal and spatial variations of the electric field. The leader channel is modeled using the non local thermo-equilibrium equations. Furthermore, statistical delays before the inception of the first corona, and random distributions to represent the tortuous nature of the path taken by the leader channel were included based on the behavior observed in experimental tests, with the intention of ensuring the discharge behaved in a realistic manner. For comparison purposes, two different gap configurations were simulated. A reasonable agreement was found between the physical model and the experimental test results. © 2013 AIP Publishing LLC. [http://dx.doi.org/10.1063/1.4818434]

I. INTRODUCTION

In the field of high voltage physics, there is still no complete understanding of how a spark traverses a large gap between a conductor and a grounded point, plate or rod. Theoretical investigations incorporating the available experimental evidence for long sparks gaps provide a means of complementing our knowledge. The majority of the experimental work has been devoted to simple arrangements such as rod–plane, rod–rod, sphere–plane among others, subject to positive switching impulses with a voltage waveform of rise times that varied between tens of microseconds to circa 500 μs and the time to half value of the voltage tail changed between 2000 μs and 5000 μs; the applied voltage could vary between hundreds to thousands of kilovolts. The distance between the electrodes changed between 1 and 16 m. All the tests have been performed at standard atmospheric conditions (20 °C and 760 mm Hg).

The design of high voltage apparatus, the coordination of the insulation, and the protection of transmission lines against overvoltages are based on knowledge of the behavior of long spark gaps. Therefore, a large number of numerical models have been proposed to reproduce these experiments. The engineering models are based on empirical formulations such as the critical radius concept of Carrara and Thione and the leader intensification criterion of Petrov and Waters. These empirical models lead to improvements in the electrical design of high voltage apparatus and insulation distances; however, they are not applicable for all kinds of electric field configurations, and it was demonstrated by the experimental work performed by Les Renardieres’ Group. Based on the results from laboratory, Rizk constructed a theory to evaluate the potential at the leader channel which has been used by Refs. 20–22 to simulate breakdowns in long spark gaps. Other models delve with physical laws, like Bondiou and Gallimberti’s model, which introduced the physic principles, and other authors’ work, like that of Lalande, Goelian et al., Castellani, Becerra and Cooray, where modifications were made to the model introduced by Gallimberti and in which lightning attachment processes or the breakdown in long gaps were calculated.

One of the most important aspects of the modeling of the discharge on long spark gaps is the streamer region. Although a great amount of theoretical and numerical studies have been devoted to streamer discharge in small gaps, it is still very difficult to simulate the dynamics and evolution of particles in the streamer region with the coupling system of the gas dynamic equations, the thermodynamic equation for molecules, and the balance equations for the number densities of various plasma species, since the extension of the streamer region usually reaches tens of centimeters.

Consequently, the assumptions underlying the streamer calculation are different in each long spark gap model, as described in the following paragraphs: Bondiou and
Gallimberti’s model calculates the charge generated by the streamer formation in terms of estimating the total number of electrons that have left the gap by reaching the high voltage electrode. The calculation takes into account the attachment process. However, as a simplification, the charge is assumed to come from a single filament, therefore, the charge for the total streamer area is estimated by multiplying the charge from a single streamer by a branching factor and by the number of filaments. In this procedure, the constants used for the first streamer and all subsequent streamers are the same.

A simplified model for the simulation of the positive spark as developed by Goelian and Lalande’s model calculates the charge associated with the advancement of the streamer-leader during a certain time instant by multiplying the area formed by the potential distribution immediately before and after the streamer formation by a geometrical constant. For the calculation, the streamer area is assumed to be comprised of \( N \) parallel lines of length \( L \) which represent the streamers, located at a distance \( a_i \) from each other. The charge simulation method is then used to evaluate the electrical charge based on the area covered by the leader between the potentials at two different instances of time and the representation of the \( N \) linear streamers.

The Becerra and Cooray streamer approximation assumes that the area covered by the streamer region required for the simulation process is conical. The charge accumulated is calculated by means of the charge simulation method. In the most recent version, Becerra and Cooray proposed a simplified method in which the area accumulated between the potential of two consecutive leader segments is proportional to the streamer charge. The proportional constant to calculate the charge is called the geometrical constant \( K \).

The three models mentioned above all presumed that the leader advances in segments according to the non-local thermo-equilibrium equation of Gallimberti. The charge per unit length required to achieve the transition to a new leader segment in the case of the models of Bondiou and Gallimberti and Becerra and Cooray is based on the thermodynamic analysis of the transition region where the streamer converges on the leader tip. The models of Lalande, Goelian, and Arevalo assumed the leader charge to take one of the values that have been reported from laboratory-based measurements, between 40 and 65 \( \mu C/m \). The magnitude used in the present calculation was based on laboratory measurements as well.

The models mentioned have been validated for configurations of a rod and a grounded plane under positive switching impulses, and the authors affirmed that the models developed can be used for sparks crossing long distances over gaps of different configurations. Part of the work reported in these references has been used to improve the understanding of the physics of the lightning attachment process, and its applications in the design of lightning protection systems.

All the models described previously assumed either a constant geometry for the estimation of the streamer region or calculate the charge accumulated based on a numerical constant neglecting the effect of the variation of electric field due to the switching impulse applied voltage on the streamer inception and propagation processes. With the aim of reducing the number of parameters assumed in the existing models, a three-dimensional identification of the streamer region was performed during every time step, considering the spatial and temporal variation of the electric field generated by the switching impulse voltage and the leader channel. The criterion used to identify the region was the called “stability field \( E_{st} \)” It has been shown from the theory that the mean electric field along the streamer filaments is about 400–500 kV/m, i.e., 450 kV/m. The model calculates the advancement of the leader channel in segments using the equation for the gradient developed by Gallimberti.

The calculation of the streamer region is based on the work presented in Ref. 28, and the complete model implemented on this paper is a continuation of the work presented by the authors in Refs. 21, 22, and 27 for positive and negative leader discharges. The model is restricted to positive switching impulse voltages as voltage source; natural conditions that could affect the inception of streamer and propagation of leader as wind, rain, or snow are not considered. The effects such as detachment, secondary emission as field emission, photoionization, or vibrational energy of molecules from the point of view of physics of particles that could change the conditions of inception of stable leader are disregarded.

To determine the effectiveness of this new model, the models of Becerra-Cooray and Lalande and the simplified alternatives put forward by these authors were implemented and compared with rod–plane arrangements tested by Paris.

II. MODEL

The main steps that are included in the model are the following:

1. Formation of a streamer corona discharge at the tip of the high voltage electrode.
2. Transformation of the stem of the streamer into thermalized leader channel (unstable leader inception).

The simulation process finishes when the streamer region reaches the grounded point, the stage at which a breakdown is considered to have occurred. If, for any reason, the streamer region seems to stop propagating, the process is assumed to have reached a “withstand voltage.” Owing to the complexity of electric field calculations, software for a finite element method was employed. The procedure proposed to evaluate the leader inception and propagation is as follows:

(a) Once the high voltage electrode is stimulated by a switching impulse voltage, the electric field produced by the voltage source is calculated, called as “background electric field.” The electric field is scanned and the probability of inception and electron in the region (see Sec. II C) and the streamer criterion are
The streamer inception criteria is calculated based on Eq. (1), considering that the phenomena is taking place in air at atmospheric standard conditions:

$$\exp \left[ \int_{\Delta x} (\alpha - \eta) \cdot dx \right] \geq N_{\text{stab}}, \quad (1)$$

where $\alpha$ is the first ionization coefficient, $\eta$ is the attachment coefficient, and $\Delta x$ is the size of the active region where $(\alpha - \eta) > 0$ and $N_{\text{stab}}$ is the minimum charge that produces a space charge field high enough to reproduce the streamer tip. The ionization and attachment coefficient are calculated as function of the local fields.$^9$

(b) The calculation starts when the streamer inception criterion is fulfilled (Eq. (1)).$^9,13$ If the streamer criterion is not fulfilled, the positive switching impulse continue increasing the magnitude of the voltage until the criterion is eventually fulfilled, at which point, a streamer started from the high voltage electrode. The charge associated with this streamer is calculated using the method described in Sec. II.A. If the charge on the streamer region $\Delta Q(t)$ is higher than 1 $\mu$C after the first streamer had been emitted,$^2,13$ a new leader, called an “unstable leader,” is initiated, i.e., a leader segment is included in the simulation to represent this stage. If the charge of the streamer corona $\Delta Q(t)$ is lower than 1 $\mu$C, unstable leader inception condition is not fulfilled and the analysis is repeated in the next time step. The unstable leader criterion of 1 $\mu$C is the charge required to increase the temperature of the discharge at the stem of the streamer to 1500 K and consequently thermalize it, creating the first leader channel. Its magnitude is based on laboratory measurements$^1$ and theoretical calculations performed by Gallimberti.$^{13}$

(c) If the condition for “unstable leader” inception is fulfilled an iterative analysis of the leader propagation starts with an assumed initial leader length of $L_{l(t0)}$ as input. The extension of the leader and the switching impulse voltage source change the potential distribution. The streamer charge generated during the extension of the leader is calculated as before but now including both the leader and its streamer region. This is facilitated by representing the drop of potential a long the leader channel $\Delta U_{l(t)}$ during the current simulation step $t$ with the non thermoequilibrium equation of Gallimberti.$^{13}$ Once the charge in the streamer region has been calculated, the advance of the leader $dl_L$ can be determined by integrating the velocity of the leader

$$dl_L = \frac{\Delta Q(t)}{q_L}; \quad (2)$$

where $q_L$ is the charge per unit length required to transform the streamer located in the active region in front of the already formed leader channel into a new leader segment. The magnitude of this $q_L$ is based on the observations made by Les Renardieres’ Group$^1$ and the used by numerical models.$^9,10,12$ As soon as the advance of the leader $dl_L$ has been calculated the new length of the leader segment $L$ of the time $(t + dt)$ can be calculated following Eqs. (3) and (4). Equation (3) considers the random characteristic of the path of the discharge, i.e., its tortuous behavior and determines the position of the leader segment in the plane, where $\phi$ is the angle of the sagitta of the segment

$$dl_L = dl_L \cdot \cos \phi, \quad (3)$$

and

$$L_L(t + dt) = L_L(t) + dl_L, \quad (4)$$

(d) Once the new charge has been calculated, the next leader segment can be inserted and the calculation to identify the streamer region starts again. If continuous propagation of the leader is observed, then it is said that the stable leader inception stage had begun. Otherwise, the leader stops and the analysis is repeated again keeping track of the space charge already developed.

A detailed explanation of the different procedures used in the model is presented in the following paragraphs.

### A. Streamer region

As it was demonstrated in experimental reports on positive discharges,$^1,6,9,13,23$ the streamer region is located at the head of the leader channel and the leader and the streamer region propagate continuously. Therefore, for each time step, both the potential drop caused by the leader channel and the change in potential arising from the streamer region are computed.

The aim of the calculation of the streamer region is to use a simplified method which can take into account the spatial and temporal variations of the electric field. The calculation does not consider in detail the solution of the continuity equation of particle densities such as ions and electrons coupled with Poisson equation. For the calculation of the streamer region, the method presented in Refs. 27 and 28 is used. The calculation of the streamer region divides up into five specific steps:

1. A point charge at the tip and finite lines of increasing length were used to simulate the leader channel. All electric field and potential calculations are solved using finite element methods. The boundary conditions for each calculation are the applied potential by the voltage source, the leader gradient of potential, the potential gradient of the previous streamer region.

2. The major assumption of the approach is that the streamer region is characterized for an average electric field of 450 kV/m. Once the potential distribution is calculated considering the background electric field and the leader channel using finite element methods. The streamer inception criterion is evaluated. If streamers are inceptioned, the 3D region where the condition that the average of the electric field between the tip of the leader and the evaluated point is 450 kV/m is identified.

3. To be able to calculate the total charge located in the region, it is assumed that the streamers propagate along the electric field lines and that the average electric field...
along each electric field line from the found region is equivalent to 450 kV/m.9,13

4. The streamer region is then divided into several layers and the charge is calculated for each layer. This calculation requires that the direction of the electric field vector be taken into consideration, including at the edge of the region, as illustrated in Figure 1.

5. Using Gauss’s theorem, the streamer charge located in each layer \( Q_j \) is calculated, and the total charge accumulated in the volume for each time step \( \Delta Q^{(j)}_{\text{total}} \) is computed. Similar method has been successfully used by Maslowski and Rakov for the study of corona shield of lightning channel:25

\[
Q_j = \int_s \varepsilon_0 \cdot \vec{E} \cdot dS,
\]

\[
\Delta Q^{(j)}_{\text{total}} = \sum_{j=1}^{n} Q_j,
\]

where \( \varepsilon_0 \) is the permittivity of air, \( \vec{E} \) is the electric field on closed surface \( S \), \( Q_j \) is the charge at each layer, and \( Q^{(j)}_{\text{total}} \) is the total charge in the \( j \)-th time step formed by the charge in the \( j \) layers which change from 1 to \( n \) and \( n \) is the total number of layers. The gauss surface consist of a closed cylindrical surface that changes for every \( j \)-th layer. The cylinder surrounded the \( j \)-th region found when condition (1) is evaluated, with an infinitesimal height. The surface \( S \) consists of the lateral surface of the cylinder, which is chosen to be such that it coincides with the outer boundaries of the streamer region. The change in the shape of the streamer region under a switching-like voltage impulse in the simulations is caused by the increment of the applied voltage to the high voltage electrode. When the voltage is increased, the electric field rises, and the region that will fulfill the condition for a streamer region grows. Therefore, the frequently made assumption of a constant geometry for the streamer region during an increment in the electric field disagrees with the real physical process of the discharge.

B. Leader channel criteria

The simulation of the leader channel made here is based on Gallimberti’s concept of “non-local thermodynamic equilibrium.”13 The leader was divided into elementary segments with length \( dl_i \), temperature \( T \), pressure \( P \), and molecular density \( n \) that is uniform along the channel. Therefore, the potential drop \( \Delta U_L \) in the segment \( i \) will be:

\[
\Delta U_L = E_{L_i} \cdot dl_i,
\]

where \( dl_i \) is the length of the segment \( i \) and \( E_{L_i} \) is the potential gradient of the segment of length \( i \).

Gallimberti’s model gives the evolution of the internal electric field \( E_{L_i} \) as a function of the injected current, assuming that the conductivity of the leader channel is essentially controlled by electronic collisions between neutral molecules and accelerated electrons in the electric field \( E_{L_i} \). It assumes that all the injected current \( I_L \) is used to dilate the leader, and that the mass remains constant as the expansion takes place. The formulation describing this hypothesis results in the following set of equations:

\[
\pi \cdot a_i^2 \cdot n_0 \cdot dl_i = \pi \cdot a_i^2 \cdot n_i \cdot dl_i,
\]

\[
\frac{\gamma - 1}{\gamma} \cdot \frac{d(\pi \cdot a^2)}{dt} = E_{L_i} \cdot I_L,
\]

where \( a_i \) is the radius of the leader segment at a defined instant in time; \( n_i \) is the neutral molecules’ density at the same time; and \( d_{0i} \) and \( n_0i \) are the initial conditions for the leader formation. \( P_0 \) is the atmospheric pressure, \( \gamma \) is the ratio between the specific heat at constant pressure and the volume constant, \( d(\pi a^2) \) is the variation in the section of the leader, and \( E_{L_i} I_L \) is the power injected into the channel during the time step, \( dt \).

By applying Eq. (8), it is possible to calculate the channel section for the next time step from the function by considering the function at the time \( t \), and the internal electric field and the charge \( I_L dt \)

\[
\pi \cdot a_i^2 \cdot (t + dt) = \pi \cdot a_i^2(t) + \frac{\gamma - 1}{\gamma} \frac{E_{L_i} \cdot I_L}{P_0} dt.
\]

As the mass is constant, the molecules’ density enables us to write

\[
n(t + dt) = n(t) \frac{\pi \cdot a_i^2(t)}{\pi \cdot a_i^2(t + dt)}.
\]

And, using the hypothesis that \( E_{L_i}/n \) is constant,\textsuperscript{13} the internal electric field at that time will be equal to

FIG. 1. Streamer region found checking condition number (1) at streamer inception instant for a rod-plane arrangement. The arrows indicate the direction of the electric field which is the direction of propagation of the streamers. The boundary condition of each line is that the electric field along it is constant and equivalent to 450 kV/m.
\[ E_{Li}(t + dt) = \frac{n(t + dt)}{n(t)} E_{Li}(t). \] (11)

Therefore, it is possible to calculate the time evolution of the internal electric field for each segment and the potential drop along the leader channel

\[ \Delta U_L = \sum_{i=1}^{k} E_{Li} \cdot dl_i, \] (12)
where \( k \) is the total number of segments.

The current injected to the head of the leader corresponds to the net charge \( \Delta Q_{\text{total}} \) associated to the advance of the streamer between times \( t \) and \( t + dt \), it can be calculated as

\[ I_L = \frac{\delta Q_{\text{total}}}{\delta t}. \] (13)

C. Statistical time delay and tortuous characteristics of the path

Electrical discharges are characterized by their random nature, taking tortuous paths and having an unpredictable inception time. These delays can be represented as statistical distributions. One statistically dependent source of delay corresponds to the combined effect of all of the elementary processes that the discharge needs to develop, and the other delay arises from the random nature of the path the discharge takes to reach the ground. The above-mentioned statistical delays are included to the model, as presented by Arevalo and co-workers.\(^{21}\) Greater details of the statistical approximations can be seen in Refs.\(^{21,22}\). A brief summary is presented in the following paragraphs.

The first time lag corresponds to the random condition of the streamer inception, when a primary free electron is in the vicinity of the electrode tip and is capable of initiating an ionization process. The probability distribution for inception was taken from the critical volume region proposed by Baldo for a streamer.\(^{30}\)

\[ p_i(t) = p_e(t) \cdot \exp \left[ - \int_0^t p_e(t') \cdot dt' \right], \] (14)

\[ p_e(t) = k_e \cdot \frac{dE}{dt} \cdot t, \] (15)

\[ a_i = \frac{1}{2 \cdot k_e \cdot \frac{dE}{dt}}, \] (16)

\( p_i(t) \) is the probability distribution for inception of corona, \( p_e(t) \) is probability distribution for having a primary free electron in the vicinity of the electrode tip, \( dE/dt \) is the electric field rate of rise, \( k_e \) is a constant that depends on the electrode shape and atmospheric conditions, and it was calculated from measurements of Ref.\(^{1}\). The magnitude of \( k_e \) used in the present study is of 0.3 cm/kV·\(\mu\)s.

A suitable Gaussian distribution \( f(\phi) \) based on the “sagitta method” proposed by Ref.\(^{32}\) was determined and applied to represent the random behavior of the leader channel.

\[ f(\phi) = a_i \cdot e^{-\left(\frac{\phi - b_i}{c_i}\right)^2}, \] (17)

where \( a_i, b_i, \) and \( c_i \) are constants obtained from tests conducted during the normal fitting of several long gap switching impulses. The fitted function determines the position of \( i \)-th leader segment in the plane, where \( \phi \) is the angle of the sagitta of the \( i \)-th segment. A random number of the Gauss distribution is calculated. Once obtained the angle \( \phi \) with the length of the leader segment \( i \)-th \( dl \), the vertical projection \( dz \) can be computed.

Table I presents the different magnitudes of the constants in the Gaussian distribution for different kinds of tips on the high voltage electrode.

D. Input parameters

The model assumes certain initial parameters for the different physical equations implemented (1) to (17). The magnitudes of these were obtained from published experimental results\(^{1}\) and are summarized in Table II.

Other initial characteristics are the boundary conditions for the electric field calculation: the high voltage electrode stressed by the switching impulse, modeled as a double exponential source\(^{26}\) with a specific rise time, time to half of voltage tail, and maximum voltage magnitude. The grounded electrode is considered as a perfect conductor.

The model uses a simplified electrostatic approach, therefore, the time step \( dt \) does not depend on the particles collision frequency, instead the time step is kept in the range of tens of microseconds based on experimental observations for the time of inception of the first streamer and leader propagation.\(^{1,5}\) As it has been used in previous models available in the literature,\(^{4,8–10}\) the maximum time step acceptable to calculate will be determined as the reason between the velocity of propagation of the positive leader\(^{8}\) and the total distance of the gap.

III. APPLICATION OF THE MODEL

To design the external and internal insulation of different equipments, engineers base its knowledge on the behavior of the discharge depending on the type of geometry, the applied voltage, and the insulation media, among others. As elaborated arrangements are difficult to understand and generalize, basic geometries have been characterized in the high

<table>
<thead>
<tr>
<th>TABLE I. Coefficients of the tortuous Gaussian distribution with 95% confidence bounds.</th>
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<tbody>
<tr>
<td>Constant</td>
</tr>
<tr>
<td>----------</td>
</tr>
<tr>
<td>( a_1 )</td>
</tr>
<tr>
<td>( b_1 )</td>
</tr>
<tr>
<td>( c_1 )</td>
</tr>
</tbody>
</table>
TABLE II. Input parameters for the numerical simulation, the numbers in bold corresponds to those used in the simulation, and the range corresponds to the values measured by Les Renardieres.1

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Magnitude</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_{l}$ [C/m]</td>
<td>45μC</td>
<td>Charge per unit length to sustain a leader channel</td>
</tr>
<tr>
<td>$E_{s}$ [kV/m]</td>
<td>450</td>
<td>Potential gradient before a new leader segment is created</td>
</tr>
<tr>
<td>$L_{d0}$ [m]</td>
<td>$2 \times 10^{-2}$</td>
<td>Initial leader length</td>
</tr>
<tr>
<td>$E_{m}$ [kV/m]</td>
<td>450–500</td>
<td>Stable electric field inside the streamer region</td>
</tr>
<tr>
<td>$a_{0}$ [μm]</td>
<td>25</td>
<td>Initial channel radius of a newly created segment</td>
</tr>
<tr>
<td>$k_{s}$ [cm/kV·μs]</td>
<td>0.3</td>
<td>Constant to define the standard deviation of the probability distribution for inception of streamer</td>
</tr>
</tbody>
</table>

...the basic electrode configuration is the rod-plane arrangement. However, there are also other kinds of arrangements that are important to understand, such as rod and a grounded rod, or a conductor and a grounded rod.

Therefore, aiming to validate the capacity of the model described in this paper to calculate the breakdown for different electrode configurations, a rod–plane and a rod-rod arrangement were simulated. In addition, calculations using other published models were performed. The implemented models were the charge simulation method of Becerra and Cooray,12 the simplified model of Goelian and Lalande,11 the simplified model of Arevalo et al.,22 and the model presented in this paper.

The authors of this paper believe that the shape of the streamer region and consequently the charge accumulated in the region depend on the characteristics of the set-up, and in particular, on the geometry of the electrodes whether this comprised two rods or a rod and a plane.

The simulated geometries are the ones tested by Paris.2 As the results obtained by Paris correspond to 50% of the breakdown voltages, it was necessary to include the statistical delay in the inception and the tortuous characteristics of the channel in all the simulated models.11,12 The statistical distribution used for all the models is the one presented in Sec. II C. To calculate the 50% breakdown voltage, 50 switching impulses were applied following the well known up and down methods.33

All the data presented here were obtained by means of the implementation of the models conducted by the authors of this paper.

A. Rod–plane gap arrangement

To validate the implemented models the rod-plane arrangement used by Paris was simulated.2 The set-up consisted of a square rod with a base area of 1 cm² and a variety of gap distances from 2 to 6 m; the switching impulse voltage waveform was characterized for a rise time of 120 μs and the time to half value of the voltage tail was 4000 μs; the maximum applied voltage was varied between 700 kV and 1600 kV.2 The aim of the test was to calculate 50% breakdown voltage, therefore, the up and down methods described in Ref. 33 were applied during testing. The simulation results obtained using the different models are presented in Figure 2.

Figure 2 shows that all of the models simulated follow the same tendency as the curve measured by Paris; however, there are differences in the 50% breakdown voltage magnitude between the measurements and the simulations. A greater difference of the 50% breakdown voltage magnitude are observed with the models proposed by Becerra and Cooray12 and Goelian and Lalande,11 which are of the order of 27% and 35%, respectively, lower than the measured magnitude. The simplified model of Arevalo et al.21 approximates the measurements of Paris closely, even though engineering simplifications have been used, the model has an absolute error of 11%, but the magnitudes of the breakdown voltages are higher than the ones measured by Paris.2 The model described and proposed in this paper has a 12% error, and the magnitudes obtained for the breakdown voltages are lower than the measured by Paris.

The results indicate that the model of Arevalo et al.21 and the one presented on this paper have almost the same percentage of error. The design of clearance distances and insulation of high voltage equipment is based on the magnitude of the 50% breakdown voltage. Consequently, a conservative value will provide a more reliable design and a higher breakdown voltage. To select higher 50% breakdown voltage as design parameter will risk the apparatus and/or station to undesirable breakdown. The model presented on this paper calculates lower breakdown voltages for the same gap distance than the one presented in paper21 and the measured in the laboratory,2 i.e., it is more conservative and reliable from the electrical design point of view. In addition, the margin between the measurements and the value obtained with the model (12%) is acceptable from the engineering point of view, and will reduce the probability of flashover of the equipment.

In general, the results obtained with all models indicate that even though the 50% breakdown voltage and the trend...
of the breakdown voltage vs. gap distance for a rod plane arrangement can be reproduced using the models presented in the literature, a fairly average degree of accuracy could be obtained. The model presented in this paper gave lower error between the measurements and the simulations and the trend of the breakdown voltage vs. gap distance is well reproduced.

### B. Rod-rod gap arrangement

The grounded rod had a height of 6 m to ground, the rod located in the high voltage side was of 6 m length and both had a base area of 1 cm². The gap distances were varied from 2 to 6 m. The switching impulse voltage waveform was characterized for a rise time of 120 μs and the time to half value of the voltage tail was 4000 μs; the maximum applied voltage was varied between 800 kV and 1700 kV. The aim of the test was to calculate 50% breakdown voltage, therefore, the up and down method described in Ref. 33 was applied during testing.

As it has been observed in Paris measurements, the height of the grounded rod will influence the breakdown voltage. Because the grounded rod will be exposed to a concentration of electric field, it is possible to incept negative streamers from the grounded electrode.

Consequently, this calculation considered two different cases: the first one does not take into account the electrical charge that can be accumulated on the grounded electrode (without checking the negative streamer inception condition, i.e., that the electric field is high enough to generate streamers on the grounded electrode, namely, that the electric field is $\geq 750 \text{kV/m}$) and the second takes into account the criterion of negative streamer inception in the rod grounded electrode.

To calculate the negative streamer inception and the geometrical shape of the charge accumulated in the grounded electrode, the model proposed in papers was used. The maximum distance the streamer charge has reached towards the high voltage electrode is calculated and once the downward coming positive leader–streamer meets the upward going negative streamer, it is considered that breakdown has occurred and the simulation will stop.

Figure 3 presents in black stars the results obtained using the simplified model proposed by Arevalo et al., such a model considers a constant of proportionality times an area formed by the potential distribution to calculate the accumulated charge at the streamer region and simulates the leader channel using Rizk equation. The black squares represent the results after the implementation of Goelian and Lalande’s methodology, which assumes the charge of the streamer region proportional to an area formed by the potential distribution and the leader channel is evaluated using Gallimberti’s equation. The black diamonds are the results obtained using the simplified method of Becerra and Cooray, which considers the charge accumulated proportional to a determined area formed by the potential distribution and the leader channel is assumed as Gallimberti’s equation. Using the mentioned methodologies, the 50% breakdown voltage for a rod-rod configuration fully disagrees with the measurements performed by Paris. The 50% breakdown voltage difference is higher than 80% and the trend of the breakdown voltage vs. distance is insensitive to a change in gap distance.

The results obtained for the methodology described in this paper are in two different colors yellow and green lines. The yellow line with circles corresponds to the case that disregarded the charge formed on the grounded rod. It is observed that the trend of the breakdown voltage vs. distance follows the same changes measured by Paris. The magnitude of the breakdown voltage is circa 20% lower than the data reported by Paris in Ref. 2.

The green line with circles represents the breakdown voltage considering that there is inception of negative streamers at the grounded electrode. The breakdown voltage magnitude is around 9% lower than the measurements reported by Paris. Consequently, one can conclude that the model that takes into account a variable streamer region can represent better the measurements of a rod-rod arrangement. To obtain more precise results, for configurations with rod electrodes grounded, the inception of negative streamers should be evaluated.

In general, the simulation of a rod-rod discharge has added the advantages of using a model that considers a variable streamer region to simulate different kinds of electrode configurations than the models that consider the streamer as a constant geometrical region and/or approximate its charge proportional to a numerical constant.

### IV. CONCLUSION

A consolidate physical methodology to evaluate the breakdown voltage under positive switching impulses for different electrode configurations is presented in this paper. It differs from the models available in the literature in such a way that the charge injected by the streamer to develop the leader and propagate it, does not depend on a fixed geometrical region or a constant of proportionality, but it is calculated for every spatial and temporal change of the electric field taking into account the variation of the applied voltage, the gap geometry, the air conditions among others, in the entire 3 dimensional region where the gap is located. In addition, the model includes the tortuous path of the leader channel and statistical delays due to the inception of corona, based on laboratory tests for different kinds of electrodes.
It is demonstrated with calculations of the 50% breakdown voltage and comparison with existing models that the assumption of a constant geometry for the streamer region will lead to erroneous values of breakdown voltage for configurations different than a rod–plane. It is proved that the streamer region will depend not only on the applied voltage but also on the electrode configuration, i.e. on the electric field conditions and consequently, the streamer geometrical form and its charge cannot be generalized for all kind of electrode geometries as it has been assumed on several existing models.

The model has been validated and compared for different configurations with successful results, and acceptable errors of less than 10% on average. Indicating the methodology can be used as a help tool to design high voltage tests and/or clearance distances.