A Comparative Study of a Sequential and Simultaneous AC-DC Power Flow Algorithms for a Multi-Terminal VSC-HVDC System

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Abstract—A comparative evaluation of a simultaneous and a sequential ac-dc power flow algorithms for a multi-terminal voltage source converter (VSC) based high-voltage direct-current (HVDC) transmission system is presented in this paper. Approximate converter losses are taken into account, using a simplified converter loss model. The algorithms are tested with different modes of VSC operation. The power flow solution is verified with the steady-state operating point in PSCAD.

Index Terms— HVDC, VSC, power flow algorithm, slack bus.

I. INTRODUCTION

With the increase in energy demand all over the world, the need for harvesting power from renewable sources and transmitting them efficiently over a long distance have given rise to several new challenges for existing alternating-current (ac) grid systems. In this context, the concept of high-voltage direct-current (HVDC) grid is emerging which can overlay on the existing ac grid and help in integration of bulk amount of renewable energy. The independent control of active and reactive power, black start capability along with some other advantages make voltage source converter (VSC) based HVDC the most suitable candidate for multi-terminal HVDC transmission [1].

For successful operation of a combined ac-dc grid, it is very important to have the capability of a very fast and accurate determination of the steady state operating point using an efficient power flow algorithm. The operating point is used for planning, scheduling and various analyses such as contingency analysis, state estimation, small signal stability analysis etc. The power flow analysis of a combined ac-dc system with thyristor based HVDC links has been studied by several research in the past [2]-[3]. With the emergence of VSC-HVDC transmission based dc grid concepts, the power flow of combined ac-dc grid has gained renewed importance. A review of the very recent literatures in this area has clearly shown that there exist two different approaches for solving the power flow of combined ac-dc grid — the sequential approach [4] and the simultaneous approach [5]. In sequential approach, the power flow of the ac system and the dc system are solved sequentially, i.e. one after another; whereas, in simultaneous method, the power flow of the entire ac-dc system is solved altogether. As an extension of the application of simultaneous method, a multi-option power flow approach for combined AC-DC grids is proposed in [6], where power flow for unified AC-DC grid and asynchronous AC grids connected via a common DC link is solved.

In this paper, a fair comparison of both sequential and simultaneous power flow algorithms is carried out mainly in terms of accuracy and complexity of execution. The rest of the paper is organized as follows: In Section II, the steady-state modeling of VSC including converter losses and the converter operating modes are discussed. The fundamentals of sequential and simultaneous load flow algorithms are discussed in Section III. The test system used for validating the algorithms is discussed in Section IV. The results of various cases and the comparative evaluation of the two algorithms are presented in Section V. Finally, conclusions are draws in Section VI.

II. VSC-HVDC CONVERTER REPRESENTATION FOR LOAD FLOW STUDIES

A. Converter station representation

In a VSC-HVDC transmission based multi-terminal system; the VSC forms the link between ac and dc systems. The VSC can be modeled as a controlled voltage source behind impedance as shown in Fig. 1. The basic assumptions used in this representation are: the phase voltages in each phase of the converter bus are symmetric and harmonic free and the converter operation is balanced. For the sake of simplicity, filters and their losses are not taken into account.

B. Converter operating modes

In a multi-terminal VSC-HVDC transmission system with \( N \) dc buses, one dc bus is considered as dc slack bus. The converter connected to that bus controls the dc voltage and compensates for the dc line losses. The remaining \( N-1 \)
converters operate in active power control mode. In the ac side, apart from controlling the active power (P), the VSC can also control either reactive power (Q) or ac bus voltage (V). Therefore in N-terminal VSC-HVDC transmission, with the active power injection of all converters at the point of common coupling (PCC) being known, every station can be modeled as PQ or PV bus from ac system’s point of view depending on the control mode of the converter connected to that bus. The reference active power at all converters except the slack converter is known at PCC. The slack converter power at PCC depends on the converter station losses, losses in dc lines and the power flow in other converters.

Figure 1. Representation of VSC for power flow study

C. Converter station loss model

The model used in this paper represents converter losses by a generalized loss formula, where converter losses vary quadratically with the phase reactor current magnitude [4]. With PCC bus voltage \( V_i \), and PCC side converter power \( S_{conv} = P_{conv} \) the converter current is calculated as,

\[ I_{conv} = \frac{S_{conv}}{V_i} \]  

The converter power losses, \( P_{loss} \) is expressed as

\[ P_{loss} = a + b \cdot abs(I_{conv}) + c \cdot abs(I_{conv})^2 \]  

III. Sequential and simultaneous power flow algorithms

A. General ac-dc power flow equations

The general power flow equations for a combined ac-dc grid are as follows:

1) AC power flow equations

Power flow equation for all buses except slack bus in ac grid can be written as,

\[ P_{inj,AC} = V_i \sum_{m=1}^{nbac} Y_{im} (V_m \cos(\theta_m) + B_m \sin(\theta_m)) \]  

\[ Q_{inj,AC} = V_i \sum_{m=1}^{nbac} Y_{im} (V_m \sin(\theta_m) + B_m \cos(\theta_m)) \]  

Where \( nbac \) is the total number of ac buses, \( Y_{dc} = G + j \cdot B \) is the bus admittance, \( V_i \) is the \( i^{th} \) ac bus voltages and \( \theta_m \) is the phase difference between \( i^{th} \) bus and \( m^{th} \) bus.

2) DC power flow equations

The injected dc power to the dc network for all buses except slack bus can be obtained in a way similar to conventional ac power flow. The power injected to the dc grid from \( i^{th} \) dc bus can be written as [7],

\[ P_{inj,DC} = -V_{DC,i} \sum_{m=1}^{nbdc} Y_{DC,im} * V_{DC,m} \]  

where \( nbdc \) is the total number of dc buses, \( Y_{DC} \) is the dc bus admittance, \( V_{DC,i} \) is the \( i^{th} \) dc bus voltage.

B. Sequential power flow algorithm

In this paper, the sequential power flow approach presented in [4] has been followed with some minor modifications. In this approach, the dc side variables are used as input to solve ac side power flow and vice versa. The advantage of sequential load flow algorithm is the easy integration of dc side equations into ac load flow framework without making any changes to the existing framework. Fig. 2 shows flow chart of sequential load flow algorithm [4].

Figure 2. Algorithm for sequential method [4]

It is worth mentioning that the dc network as well as the ac network power flows has to be solved iteratively. Once the dc slack bus power injection is updated, the ac power flow solution changes. So, apart from these internal iterations for dc and ac power flow solutions, an external iteration loop is required to ensure the overall convergence of the algorithm. In [4], the initial dc slack bus power is estimated as the algebraic sum of all other converters, which can increase the total number of iterations. Instead of that, in this paper the initial dc slack bus power is estimated by running dc load flow with a flat voltage estimation of 1.0 p.u. This reduces the total number of iterations, since the actual dc bus voltages are very near to it. The slack converter power at corresponding PCC is calculated as
\[ P_{\text{slack}} = V_{DC,\text{slack}} \sum_{m=1}^{\text{nhdc}} V_{DC,m} Y_{DC,\text{slackm}} - P_{\text{loss,slack}} \]  

(6)

where \( P_{\text{loss,slack}} \) is the dc slack converter losses. With this slack power injection, the ac load flow is performed to get ac bus voltage.

The nonlinear set of ac power flow equations is solved using Newton-Raphson’s method which can be written as,

\[ \Delta M_{\text{AC}} = -J_{\text{AC}} \Delta X_{\text{AC}} \]  

(7)

where \( \Delta X_{\text{AC}} \) is the incremental vector of bus voltages and angles and \( J_{\text{AC}} \) is the ac Jacobian matrix. The power mismatch equations, \( \Delta M_{\text{AC}} \) at PCC are defined as,

\[ 0 = P_{\text{gen},j} - P_{\text{ inj,ACj}} - P_{\text{conv,j}} \]  

(8)

\[ 0 = Q_{\text{gen},j} - Q_{\text{ inj,ACj}} - Q_{\text{conv,j}} \]  

(9)

Where \( P_{\text{gen}} \) and \( Q_{\text{gen}} \) are the generated power at \( j \)th bus, \( P_{\text{ inj,ACj}} \) can be obtained from (3) and (4) and \( P_{\text{conv,j}} \) is specified at PCC if it is connected to, else it is zero. \( Q_{\text{conv,j}} \) will be specified if the converter is in PQ control mode.

With these ac voltages available from ac power flow, the converter current is calculated using (1), to determine the converter losses from (2). With the losses known, the injected power to the dc side, \( P_{\text{ inj,i}} \) is calculated using,

\[ P_{\text{ inj,i}} = P_{\text{conv,i}} - P_{\text{loss,i}} \]  

(10)

The nonlinear dc equations are solved using Newton Raphson, which can be written as,

\[ \Delta M_{\text{DC}} = -J_{\text{DC}} \Delta X_{\text{DC}} \]  

(11)

where, \( \Delta X_{\text{DC}} \) is the incremental vector of dc bus voltages

The power mismatch equations, \( \Delta M_{\text{DC}} \) are given by,

\[ 0 = P_{\text{ inj,i}} - P_{\text{conv,i}} - P_{\text{loss,i}} \]  

(12)

From the dc load flow solution, the dc bus voltages are obtained. With these new dc bus voltages the dc slack bus power is calculated using (5).

After solving dc power flow, for calculating the active power injected into ac grid by the slack converter, the slack converter station losses are required. But the station loss depends on the converter current which is still unknown. In this paper, with an initial guess of converter losses, the power injected to ac grid is first calculated using (6). Form this power, the slack converter loss is again calculated using (1) and (2). In the next iteration, this new converter loss is used to find \( P_{\text{slack}} \). The iterations are carried out till the converter loss converges.

C. Simultaneous power flow algorithm

The simultaneous approach compared in this paper is identical with [5]. In this method, both ac and dc sides are considered together as a unified ac-dc grid for solving the power flow. Since ac and dc equations are solved simultaneously, an external iteration loop is not required here. However, in this algorithm, the slack station losses are considered as a separate variable \( X_s \). Apart from ac and dc mismatch equations, an additional mismatch equation is therefore included to account for slack converter losses [5]. The additional mismatch equation is given by,

\[ 0 = X_s - P_{\text{loss,slack}} \]  

(13)

where \( P_{\text{loss,slack}} \) is obtained from (1) and (2). Fig. 3 shows flow chart of simultaneous load flow algorithm.

From the initial estimation of state variables, first the dc slack bus power at PCC is calculated from (6). The next step is to calculate \( P_{\text{ inj,ACi}} \) and \( Q_{\text{ inj,ACi}} \) from (3) and (4). Now the converter losses are calculated form (2). After this, the power injected into the dc side is calculated from (10). Now the ac and dc mismatch equations are calculated form (8), (9) and (11). The combined mismatch equation which are then solved in each iteration can be expressed as,

\[ \Delta \mathbf{P} = -J \Delta \mathbf{X} \]  

(13)

where, \( \Delta \mathbf{X} \) is incremental vector of state variables and \( \Delta \mathbf{X} = [\Delta X_1, \Delta X_2, \Delta X_3] \)

\( \Delta X_1 = \text{Incremental vector of bus voltages and angles} \)

\( \Delta X_2 = \text{Incremental vector of dc bus voltages} \)

\( \Delta X_3 = \text{Incremental vector of dc slack converter loss} \)

\( \Delta \mathbf{P} = \text{Power mismatch vector} \)

\( \Delta P = [\Delta P_{\text{AC}}, \Delta P_{\text{DC}}, \Delta P_{\text{slack}}] \)

\( \Delta P_{\text{AC}} = \text{AC power mismatch vector} \)

\( \Delta P_{\text{DC}} = \text{DC power mismatch vector} \)

\( \Delta P_{\text{slack}} = \text{Slack converter loss mismatch vector} \)

\( J = \text{combined ac-dc system Jacobian matrix} \)

In this algorithm, after every iteration state variables are updated and the iteration is continued till convergence.

IV. Test System

In order to validate the power flow code developed, Kundur’s two area systems [8] is modified with dc grid embedded in it as shown in Fig. 4. The line impedances are same as that of Kundur’s system. In the ac side, two additional generators G5 and G6 are added. Under nominal condition each of G1, G2, G4, G5 and G6 generates 700MW whereas G3 is left as slack. Additional ac lines are also included to make the dc network completely embedded in the ac network, to show the effectiveness of both algorithms. The converters connected to dc buses DC1, DC2 and DC3 control the dc power at 600MW, 300MW and 300MW respectively. The converter connected to DC4, which is the slack converter keeps the dc link voltage constant at ±320kV. For the sake of simplicity, the dc network is considered to be having a symmetric monopole configuration. The load connected to the
grid is shown in Table I. The base power for the dc side is taken as 600MW. However, the base MVA for the ac side is considered to be 100 MVA.

\[
\text{Initial estimate of state variable } X \rightarrow \text{Calculate dc slack bus power at PCC} \\
\quad \rightarrow \text{Calculate } P_{\text{inj,AC}} \text{ and } Q_{\text{inj,AC}} \\
\quad \rightarrow \text{Calculate converter losses and } P_{\text{inj,DC}} \\
\quad \rightarrow \text{Check mismatch equation} \\
\quad \rightarrow \text{Yes} \rightarrow \text{Output} \\
\quad \rightarrow \text{No} \rightarrow \text{Converged?} \\
\rightarrow \text{Update } X \\
\]

Figure 3. Algorithm for simultaneous method [5]

<table>
<thead>
<tr>
<th>BUS</th>
<th>LOAD</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>400</td>
</tr>
<tr>
<td>9</td>
<td>2300</td>
</tr>
<tr>
<td>13</td>
<td>1400</td>
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</tbody>
</table>

Table I. Connected Load

V. RESULTS AND DISCUSSIONS

A. Results

This section presents results of the sequential and simultaneous ac-dc power flow algorithms. In this implementation, a tolerance of 1e-5 p.u. was considered for convergence of both algorithms. Two cases are studied here.

Case 1: All converters in PQ mode with zero reactive power injection and with rated active power.

Case 2: All the converters are in PV mode maintaining the PCC voltage to 1.0 p.u. The power flow results using both algorithms are summarized in Table II and in Table III.

In case 1, with sequential algorithm, power flow convergence in two external iterations when a flat start is considered. In the first external iteration, ac side requires five and dc side requires two internal iterations for convergence. In the second external iteration, ac side requires three and dc side requires one internal iteration for convergence. The numbers of external iteration increase if we reduce the tolerance level. Also, the number can be reduced, if the dc slack bus power at PCC is calculated as discussed in Section III. On the other hand, with simultaneous algorithm, the power flow converges
in four overall iterations. However, it is important to note that both the algorithms converge to the same solution.

In case 2, each VSC generates reactive power independently to maintain ac bus voltage constant. With sequential algorithm, power flow converges in two external iterations when a flat start is considered. In the first external iteration, ac side requires five and dc side requires two iterations for convergence. In the second external iteration, ac side requires three and dc side requires one iteration for convergence. In simultaneous algorithm, again four overall iterations are to be carried out for convergence. In this case also, both the algorithms converge to the same solution. In order to validate the solution, the power flow results are compared with PSCAD/EMTDC simulation results in steady state as shown in Fig. 5 and found to be matching.

B. Discussions

Based on the experience of carrying out the power flow study with these two algorithms, following remarks would be worth-mentioning:

- The sequential method has the advantage that, it can be implemented easily as an extension of existing ac load flow programs, whereas simultaneous algorithm requires modification of existing ac load flow codes.
- Both the algorithms can converge to the same solution with small difference in overall number of iterations.

<table>
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<tr>
<th>DC Bus</th>
<th>Voltage (p.u)</th>
<th>Angle (rad)</th>
<th>P_{inj} (p.u)</th>
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### TABLE II. POWER FLOW SOLUTIONS - CASE 1

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<th>Voltage (p.u)</th>
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### TABLE III. POWER FLOW SOLUTIONS - CASE 2

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The sequential algorithm has two internal iteration loops (one for ac side and the other for dc side) and one external iteration loop (to make sure the overall convergence). Apparently, this increases the number of calculations. However, for a realistic ac-dc system the converter loss is a very small percentage of the system power level. Therefore, the external iteration loop, which is based on the dc slack converter loss, will not contribute much to the accuracy of the results.

In simultaneous algorithm, ac and dc equations are solved together in the same iteration. However, the Jacobian of the combined ac-dc system has to be reconstructed which makes the implementation more complex compared to the sequential method.

VI. CONCLUSIONS
In this paper, the performance of a sequential and simultaneous algorithm for power flow calculations in a VSC based combined ac-dc grid has been compared. The Matlab based power flow results are further verified with steady-state results in PSCAD. The relative advantages and disadvantages of each algorithm are evaluated mainly in terms of computational complexity, number of iterations and ease of implementation.

REFERENCES